

Electrostatics



Definition: Electricity at rest (stationary)

Static means to stand and is used in Mechanical Engineering to study forces on bridges and other structures.

Statue, stasis, stationary, ecstatic, status

Examples of Electrostatics

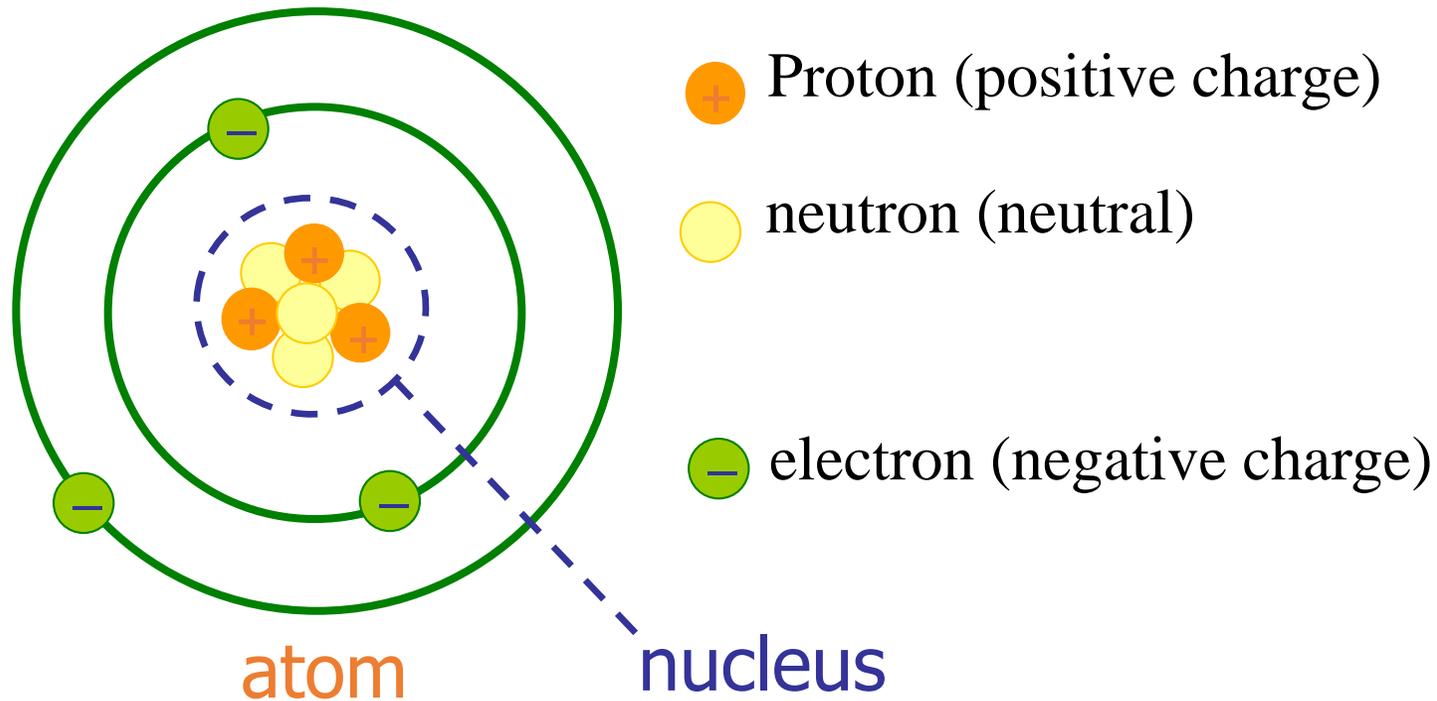
- Thunder and Lightning
- Static shocks from carpet and doorknobs
- Rubbing balloons on hair and sticking to the wall
- Rubbing Comb and attracting paper

This lecture will help you understand:

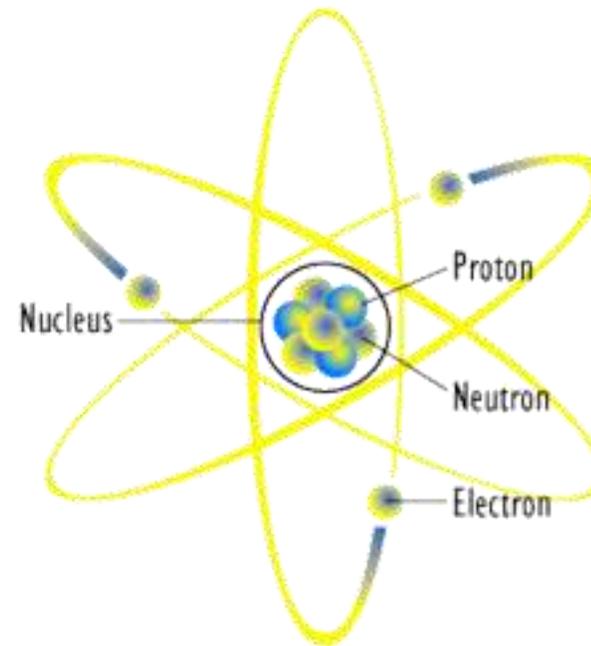
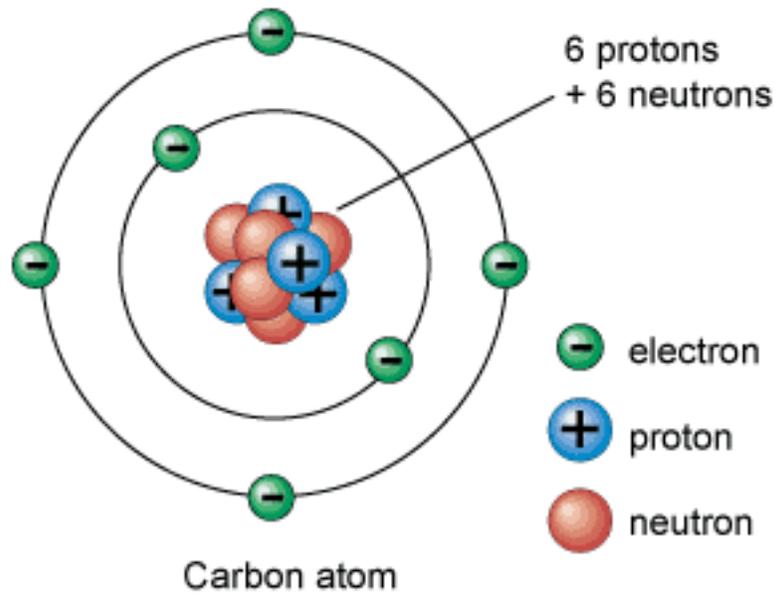
- Electrical Forces and Charges
- Conservation of Charge
- Coulomb's Law
- Conductors and Insulators
- Superconductors
- Charging
- Charge Polarization
- Electric Field
- Electric Potential
- Electric Energy Storage

Where do charges come from?

Matter is made up of **atoms**.



Model of the Atom



Electric Force and Charges

Fundamental facts about atoms

1. Every atom is composed of a positively charged nucleus surrounded by negatively charged electrons.
2. Each of the electrons in any atom has the same quantity of negative charge and the same mass.

Electric Force and Charges

Fundamental facts about atoms (continued)

3. Protons and neutrons compose the nucleus. Protons are about 1800 times more massive than electrons, but each one carries an amount of positive charge equal to the negative charge of electrons. Neutrons have slightly more mass than protons and have no net charge.

4. Atoms usually have as many electrons as protons, so the atom has zero net charge.

Electric Force and Charges

Ion

- Positive ion—atom losing one or more electrons has positive net charge.
- Negative ion—atom gaining one or more electrons has negative net charge.

Electric Force and Charges

Electrons in an atom

- Innermost—attracted very strongly to oppositely charged atomic nucleus
- Outermost—attracted loosely and can be easily dislodged

Electric Charge (q, Q)

1. Charge exists as $+q$ and $-q$. At the same point: $+q - q = 0$
2. Charge is conserved (locally).
3. Charge is quantized. $+q = n (+e)$, $-q = m (-e)$, m, n , integer

electron: $-e$, positron: $+e$, proton: $+e$, C-nucleus: $6(+e)$

Charge conservation in the micro world:

$p + e \rightarrow n$ (electron capture)

Macro world: $q \sim 10^{23}e$

Quantization is unimportant. Imagine charge as some kind of jelly.

Conservation of Charge

Conservation of charge

- In any charging process, no electrons are created or destroyed. Electrons are simply transferred from one material to another.



Charging

- Charging by friction and contact.

Example:

Stroking cats fur, combing your hair, rubbing your shoes on a carpet

- Electrons transfer from one material to another by simply touching. For example,
 - when a negatively charged rod is placed in contact with a neutral object, some electrons will move to the neutral object.

Where do charges come from?

Rubbing materials does NOT create electric charges. It just transfers electrons from one material to the other.

Electro-negativity

Relative
electro-negativity
ranking for some
common materials
from electron donating
materials (+, glass) to
electron accepting
materials (-, teflon)

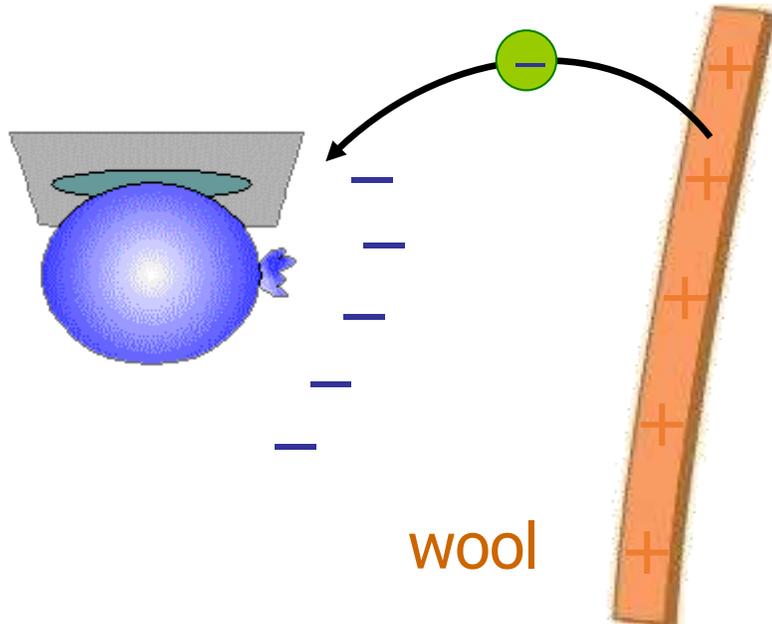
+ + + + +
+ + + +
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+ +
+

-
- -
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- Glass
- Human Hair
- Nylon
- Silk
- Fur
- Aluminum
- Paper
- Cotton
- Copper
- Rubber
- PVC
- Teflon

Where do charges come from?

When a balloon rubs a piece of wool...



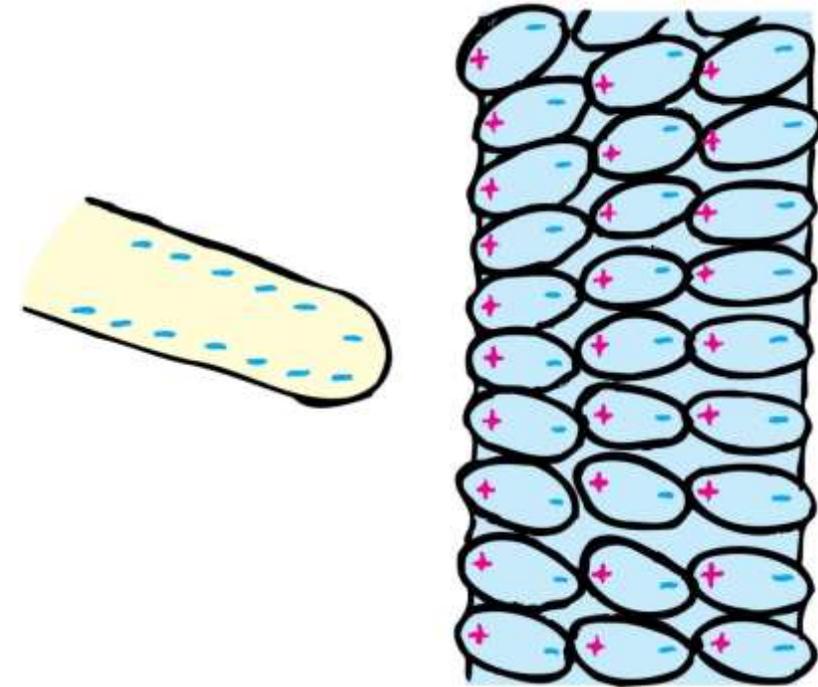
electrons are pulled from the wool to the balloon.

The balloon has more electrons than usual.

The balloon: $-$ charged,
The wool: $+$ charged

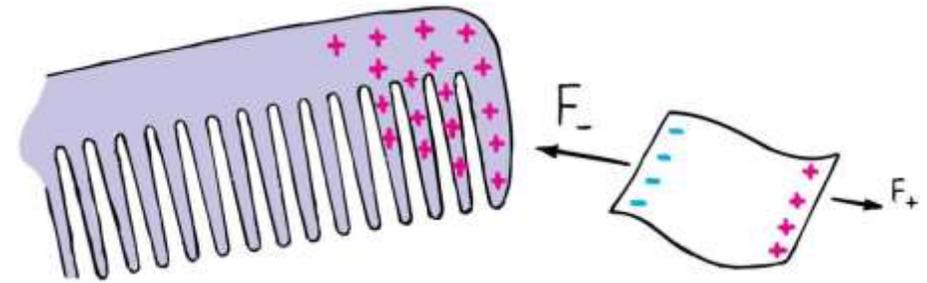
Charge Polarization

- If the charged rod is negative, then the positive part of the atom or molecule is tugged in a direction toward the rod, and the negative side of the atom or molecule is pushed in a direction away from the rod.
- The positive and negative parts of the atoms and molecules become aligned. They are electrically polarized.



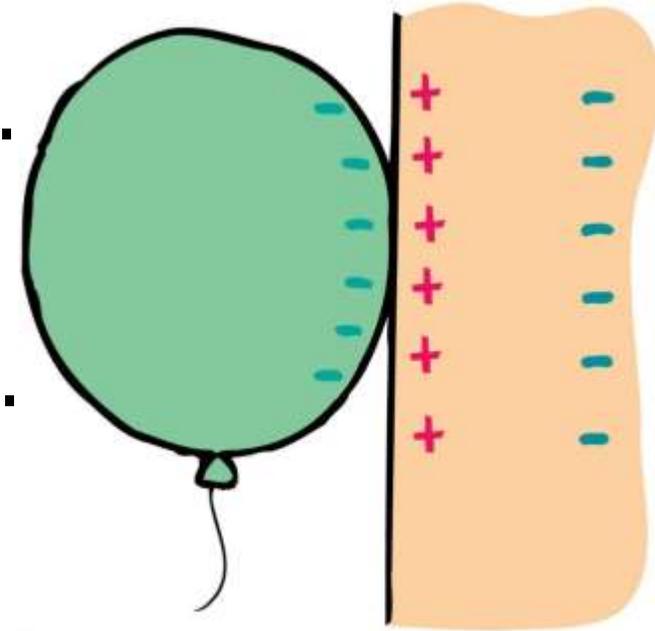
Charge Polarization

- When a charged comb is brought nearby, molecules in the paper are polarized.
- The sign of charge closest to the comb is opposite to the comb's charge.
- Charges of the same sign are slightly more distant. Closeness wins, and the bits of paper experience a net attraction.



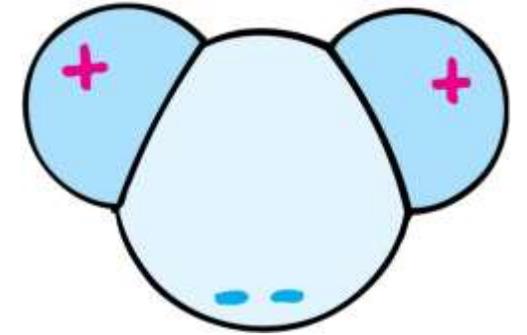
Charge Polarization

- Rub an inflated balloon on your hair, and it becomes charged.
- Place the balloon against the wall, and it sticks.
- This is because the charge on the balloon induces an opposite surface charge on the wall.
- Again, closeness wins, for the charge on the balloon is slightly closer to the opposite induced charge than to the charge of same sign



Charge Polarization

- Many molecules— H_2O , for example—are electrically polarized in their normal states.
- The distribution of electric charge is not perfectly even.
- There is a little more negative charge on one side of the molecule than the other.
- Such molecules are said to be *electric dipoles*.



Induction versus Conduction

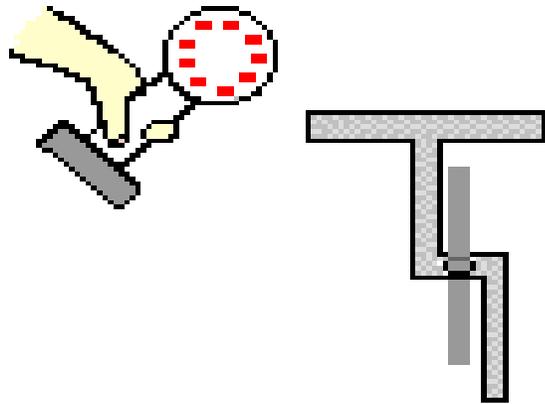
Charging by Conduction:

Transferring charges by **touching** a charged object to an uncharged one.

Example: Electroscope

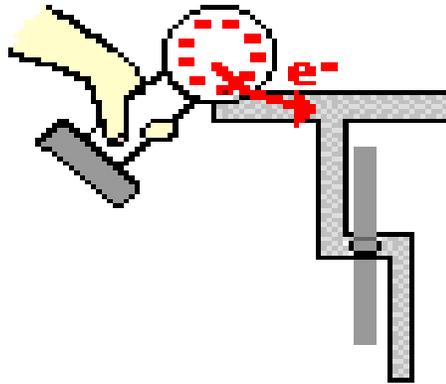
Charging a Neutral Object by Conduction

Diagram i.



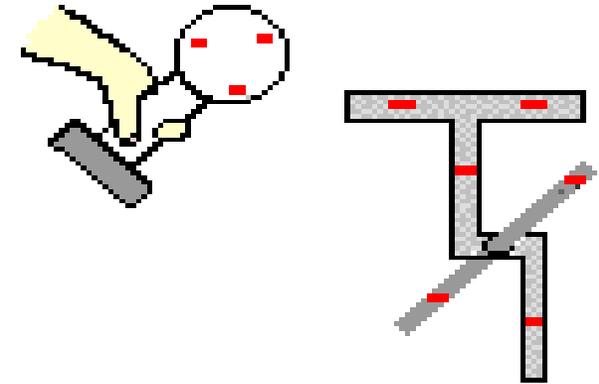
A metal sphere with an excess of - charge is brought near to a neutral electroscope.

Diagram ii.



Upon contact, e^- move from the sphere to the electroscope and spread about uniformly.

Diagram iii.



The metal sphere now has less excess - charge and the electroscope now has a - charge.

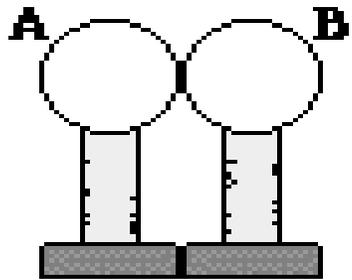
Charging by Induction:

Transferring charges *without* touching a charged object to an uncharged one.

Example: Large amounts of negative charges in storm clouds *induces* separation of charge on ground and lightning results.

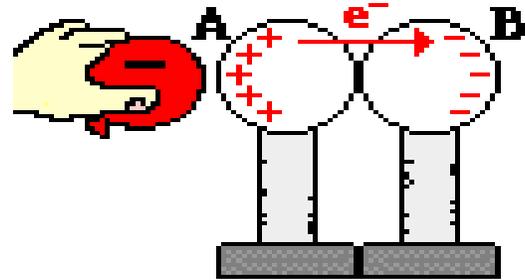
Charging by Induction

Diagram i.



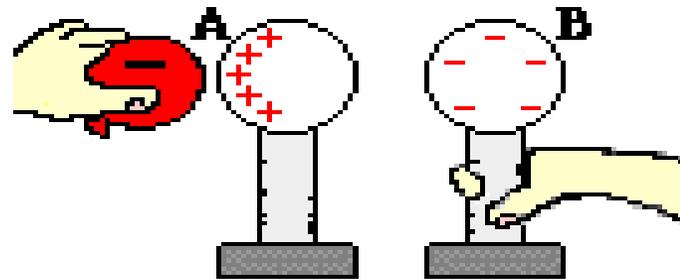
Two metal spheres are mounted on insulating stands.

Diagram ii.



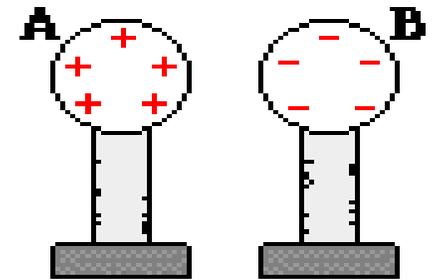
The presence of a - charge induces e^- to move from spheres A to B. The two-sphere system is polarized.

Diagram iii.



Sphere B is separated from sphere A using the insulating stand. The two spheres have opposite charges.

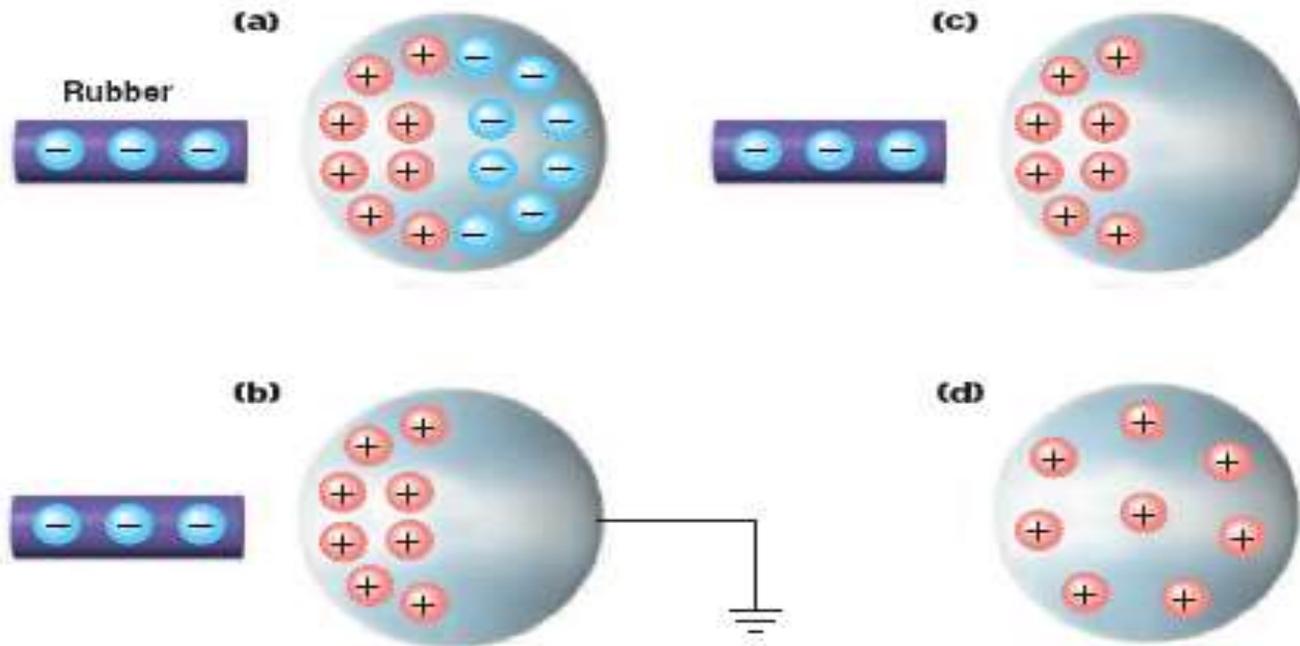
Diagram iv.



The excess charge distributes itself uniformly over the surface of the spheres.

Charge Induction

Charging by Induction



- Charging by induction
 - If you bring a charged object *near a conducting surface*, *electrons are made to move* in the surface material, even without physical contact.

– Example: The negative charge at the bottom of the cloud induces a positive charge on the buildings below.



Electric Force and Charges

Electrons in an atom

Examples:

- When rubbing a comb through your hair, electrons transfer from your hair to the comb. Your hair has a deficiency of electrons (positively charged).
- When rubbing a glass rod with silk, electrons transfer from the rod onto the silk and the rod becomes positively charged.

In Summary...

Objects become electrically charged in three way:

1. By *friction*
2. By *conduction* (contact)
3. By *induction* (no contact).

Conductors and Insulators

- **Conductor:** Materials in which one or more of the electrons in the outer shell of its atoms are not anchored to the nuclei of particular atoms but are free to wander in the material
 - **Example:** Metals such as copper and aluminum
- **Insulators:** Materials in which electrons are tightly bound and belong to particular atoms and are not free to wander about among other atoms in the material, making them flow
 - **Example:** Rubber, glass

Conductors and Insulators

- **Semiconductors:** A material that can be made to behave sometimes as an insulator and sometimes as a conductor.
 - Fall in the middle range of electrical resistivity between insulators and conductors.
 - They are insulators when they are in their pure state.
 - They are conductors when they have impurities.
- Semiconductors conduct when light shines on it.
 - If a charged selenium plate is exposed to a pattern of light, the charge will leak away only from the areas exposed to light.

Insulators and conductors

Insulators: materials that do NOT allow electrons to flow through them easily.

Insulators can be easily charged by friction as the extra electrons gained CANNOT easily escape.

4 Insulators and conductors

Conductors: materials that allow electrons to flow through them easily.

Conductors **CANNOT** be easily charged by friction as the extra electrons gained can easily escape.

Superconductors

- Superconductors: Materials acquire zero resistance (infinite conductivity) to the flow of charge.
 - Once electric current is established in a superconductor, the electrons flow indefinitely.
 - With no electrical resistance, current passes through a superconductor without losing energy.
 - No heat loss occurs when charges flow.

Coulomb's Law

- Relationship among electrical force, charge, and distance discovered by Charles Coulomb in the 18th century
- States that for a pair of charged objects that are much smaller than the distance between them, the force between them varies directly, as the product of their charges, and inversely, as the square of the separation distance

Coulomb's Law (continued)

Coulomb's law If the charges are alike in sign, the force is repelling; if the charges are not alike, the force is attractive.

- In equation form:

$$F = k \frac{q_1 q_2}{d^2}$$

$$k = 9,000,000,000 \text{ Nm}^2/\text{C}^2$$

- Unit of charge is coulomb, C
- Similar to Newton's law of gravitation for masses
- Underlies the bonding forces between molecules

2.1.1 Introduction

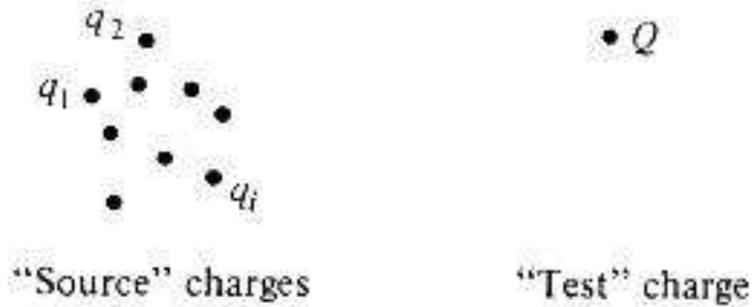


Figure 2.1

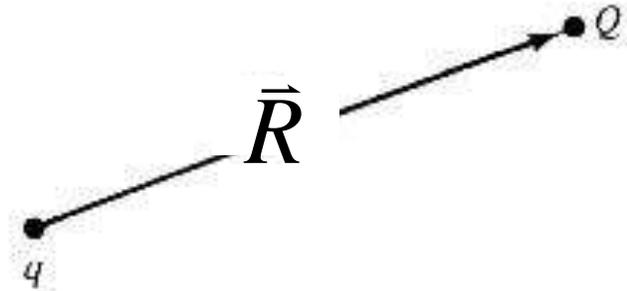


Figure 2.2

The fundamental problem for electromagnetism to solve is to calculate the interaction of charges in a given configuration.

That is, what force do they exert on another charge Q ?

The simplest case is that the source charges are stationary.

Principle of Superposition:

The interaction between any two charges is completely unaffected by the presence of other charges.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

\vec{F}_i is the force on Q due to q_i

2.1.2 Coulomb's Law

The force on a charge Q due to a single point charge q is given by Coulomb's law

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{R^2} \hat{R} \quad \vec{R} = \vec{r}_Q - \vec{r}_q = R \hat{R}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

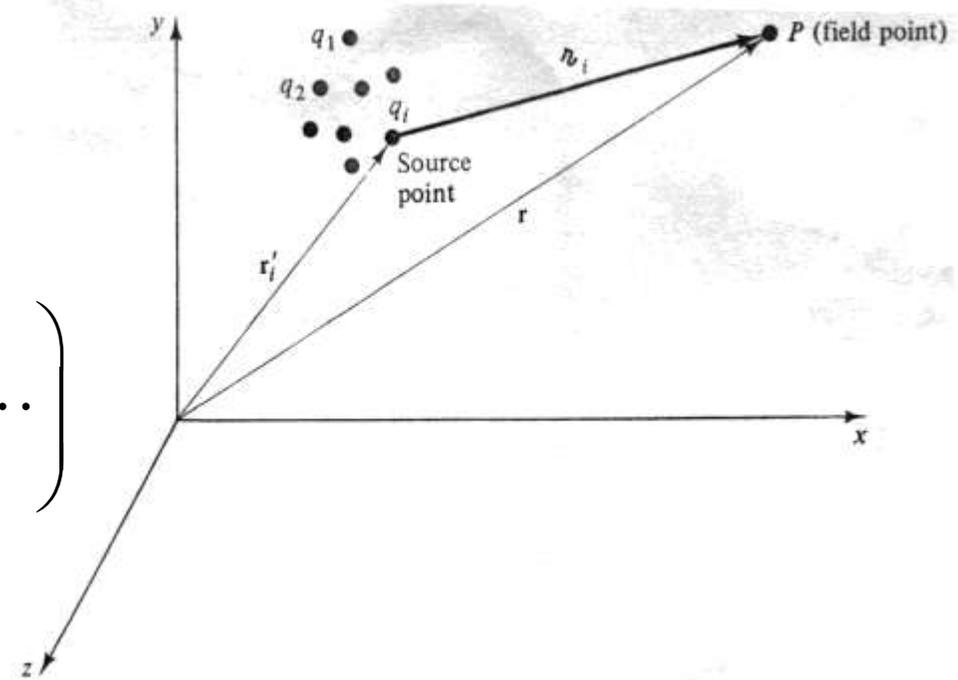
the permittivity of free space

2.1.3 The Electric Field

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots$$

$$= \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 Q}{R_1^2} \hat{R}_1 + \frac{q_2 Q}{R_2^2} \hat{R}_2 + \dots \right)$$

$$= \frac{Q}{4\pi\epsilon_0} \left(\frac{q_1}{R_1^2} \hat{R}_1 + \frac{q_2}{R_2^2} \hat{R}_2 + \frac{q_3}{R_3^2} \hat{R}_3 + \dots \right)$$



$$\vec{F} = Q\vec{E} \quad \vec{E}(P) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{R_i^2} \hat{R}_i$$

↑
the electric field of the source charge

2.1.4 Continuous Charge Distributions

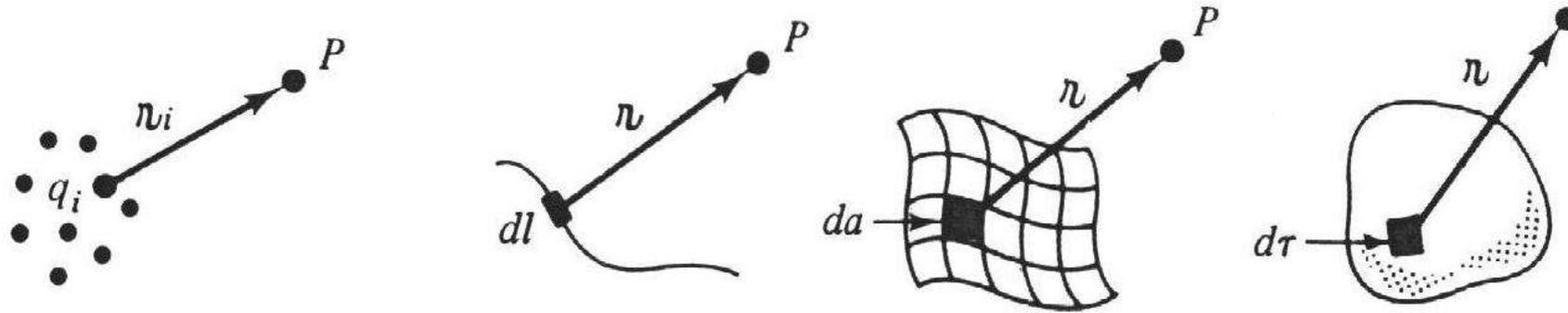


Figure 2.5

Point charges, q_i

Line charge, λ

Surface charge, σ

Volume charge, ρ

$$\sum_{i=1}^n () q_i \sim \int_{line} () \lambda dl \sim \int_{surface} () \sigma da \sim \int_{volume} () \rho d\tau$$

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{R_i^2} \hat{R}_i$$

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0} \int_{line} \frac{\hat{R}}{R^2} \lambda dl$$

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0} \int_{surface} \frac{\hat{R}}{R^2} \sigma da$$

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0} \int_{volume} \frac{\hat{R}}{R^2} \rho d\tau$$

2.1.4 (2)

Example:

the test point is at $\vec{p} = (x, y, z)$ the source point is at (x', y', z')

$$\vec{R} = (x - x')\hat{i} + (y - y')\hat{j} + (z - z')\hat{k}$$

$$\bar{R} = \left[(x - x')^2 + (y - y')^2 + (z - z')^2 \right]^{1/2} \quad \hat{R} = \frac{\vec{R}}{|\bar{R}|}$$

$$\vec{E}(x, y, z) = \frac{1}{4\pi\epsilon_0} \int_{\text{volume}} \frac{(x - x')\hat{i} + (y - y')\hat{j} + (z - z')\hat{k}}{\left[(x - x')^2 + (y - y')^2 + (z - z')^2 \right]^{3/2}} \rho(x', y', z') dx' dy' dz'$$

$$\begin{aligned} \therefore \vec{E}(x, y, z) &= \frac{1}{4\pi\epsilon_0} \int_{\text{volume}} \frac{\hat{R}}{R^2} \rho(x', y', z') dx' dy' dz' \\ &= \frac{1}{4\pi\epsilon_0} \int_{\text{volume}} \frac{\hat{R}}{R^2} \cdot \frac{R}{R} \rho(x', y', z') dx' dy' dz' \\ &= \frac{1}{4\pi\epsilon_0} \int_{\text{volume}} \frac{\vec{R}}{R^3} \rho(x', y', z') dx' dy' dz' \end{aligned}$$

2.1.4 (3)

Example 2.1 Find the electric field a distance z above the midpoint of a straight of length $2L$, which carries a uniform line charge λ

Solution:

$$d\vec{E} = 2 \frac{1}{4\pi\epsilon_0} \left(\frac{\lambda dx}{R^2} \right) \cos \theta \hat{z}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{2\lambda z}{(z^2 + x^2)^{3/2}} dx$$

$$= \frac{2\lambda z}{4\pi\epsilon_0} \left[\frac{x}{z^2 \sqrt{z^2 + x^2}} \right]_0^L$$

$$= \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z \sqrt{z^2 + L^2}}$$

(1) $z \gg L$

$$\vec{E} \cong \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z^2}$$

(2)

$L \rightarrow \infty$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{z}$$

