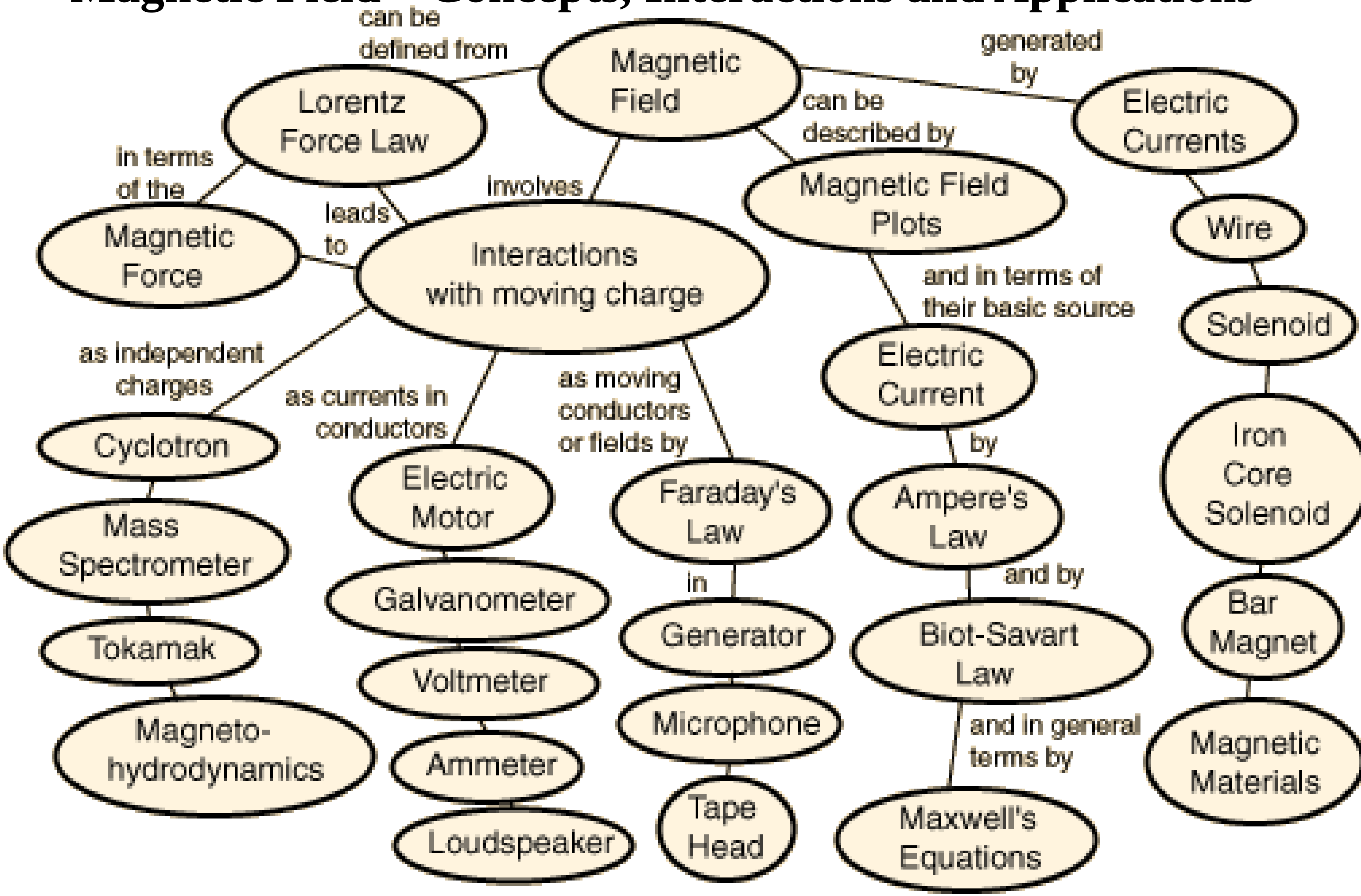
Magnetic Field

Magnetic Field – Concepts, Interactions and Applications



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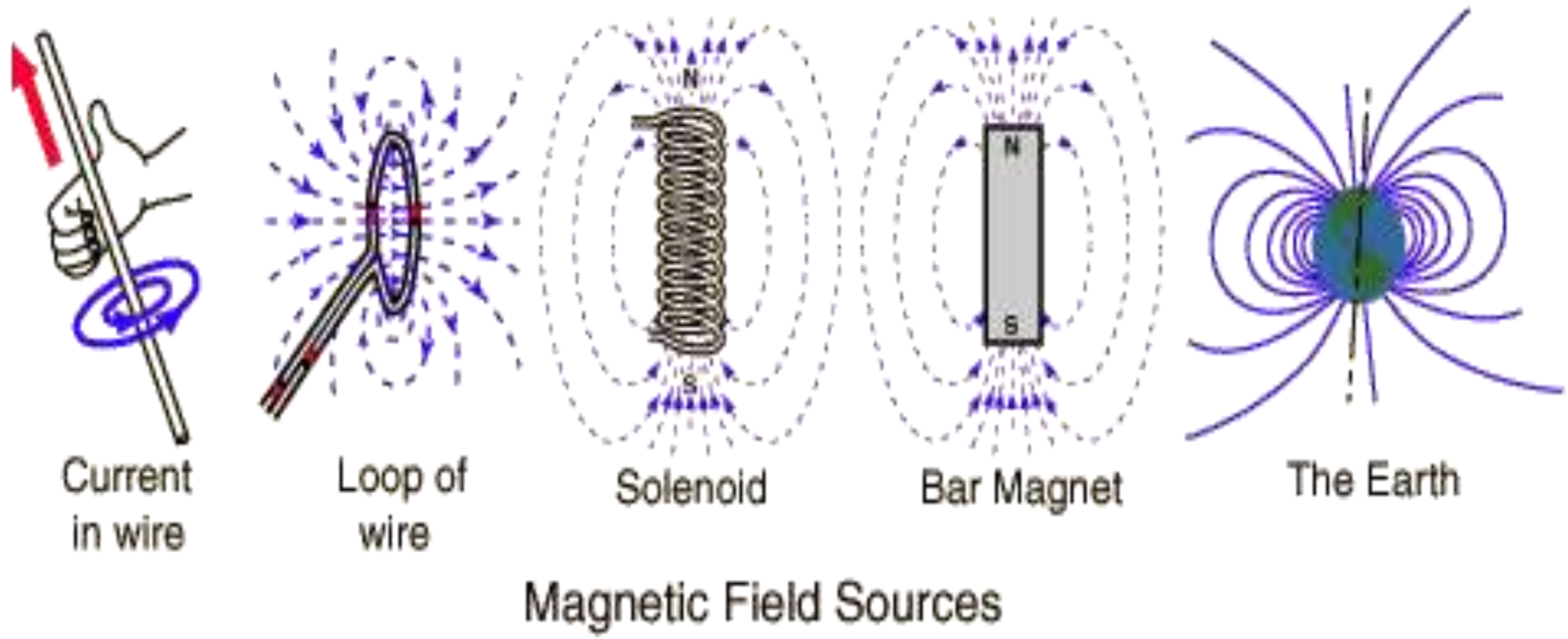
Magnetic Field Sources

Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits.



Magnetic Field due to a Long Straight Wire:

The magnetic field vector at any point is tangent to a circle.

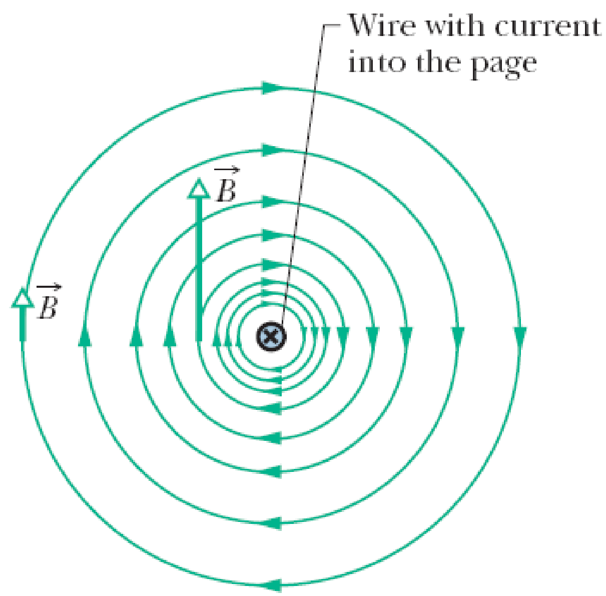


Fig. 29-2 The magnetic field lines produced by a current in a long straight wire form concentric circles around the wire. Here the current is into the page, as indicated by the ×.

The magnitude of the magnetic field at a perpendicular distance R from a long (infinite) straight wire carrying a current i is given by

$$B = \frac{\mu_0 i}{2\pi R} \qquad \text{(long straight wire).}$$

Fig. 29-3 Iron filings that have been sprinkled onto cardboard collect in concentric circles when current is sent through the central wire. The alignment, which is along magnetic field lines, is caused by the magnetic field produced by the current. *(Courtesy Education Development Center)*

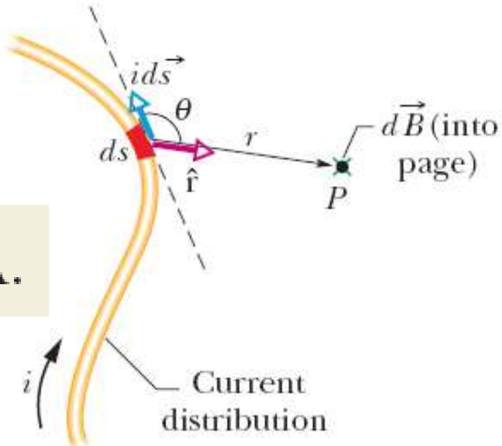
Calculating the Magnetic Field due to a Current

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2},$$

Symbol μ_0 is a constant, called the permeability constant, whose value is

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\cdot m/A} \approx 1.26 \times 10^{-6}\,\mathrm{T\cdot m/A}.$$

This element of current creates a magnetic field at *P*, into the page.



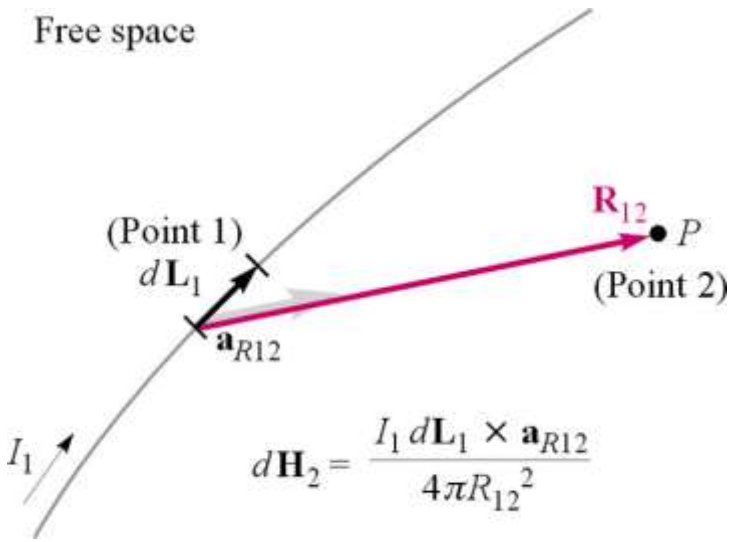
In vector form

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2} \qquad \text{(Biot–Savart law).}}$$

Fig. 29-1 A current-length element $i\,d\vec{s}$ produces a differential magnetic field $d\vec{B}$ at point P. The green $\times$ (the tail of an arrow) at the dot for point P indicates that $d\vec{B}$ is directed *into* the page there.

1820, Hans Christian Oersted, electric currents and compass

Biot-Savart Law



At any point P the magnitude of the magnetic field intensity produced by a differential element is proportional to the product of the current, the magnitude of the differential length, and the sine of the angle lying between the filament and a line connecting the filament to the point P at which the field is desired; also, the magnitude of the field is inversely proportional to the square of the distance from the filament to the point P. The constant of proportionality is $1/4\pi$

$$dH = \frac{IdL \times a_R}{4 \cdot \pi \cdot R^2} = \frac{IdL \times \vec{R}}{4 \cdot \pi \cdot R^3}$$

Magnetic Field Intensity A/m

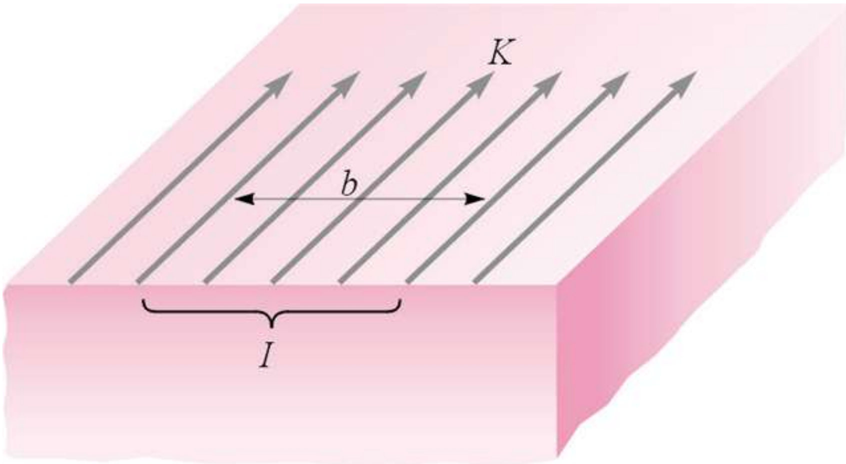
$$H = \oint \frac{I}{4 \cdot \pi \cdot R^2}\, dL \times a_R$$

Verified experimentally

Biot-Savart = Ampere's law for the current element.

Biot-Savart Law

B-S Law expressed in terms of distributed sources



The total current I within a transverse Width b, in which there is a uniform surface current density K, is Kb.

$$I = \int K\, dN$$

For a non-uniform surface current density, integration is necessary.

Alternate Forms

$$H = \int \frac{K\_x}{4\cdot\pi\cdot R^2}\, dS\cdot a_R \qquad H = \int \frac{J\_x}{4\cdot\pi\cdot R^2}\, dv\cdot a_R$$

Magnetic Field due to a Long Straight Wire:

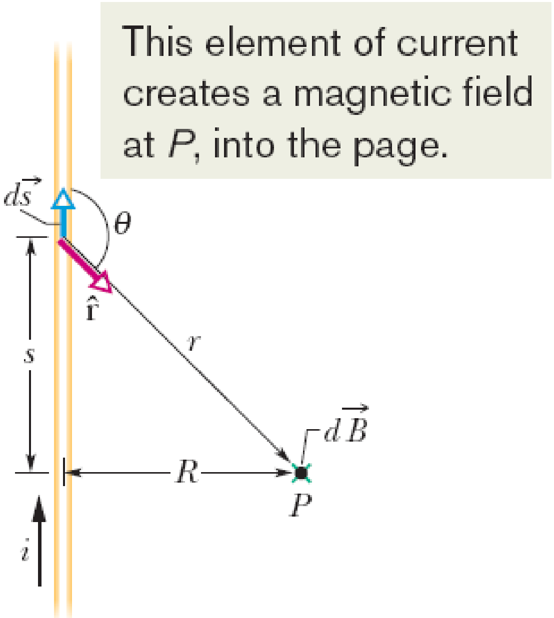


Fig. 29-5 Calculating the magnetic field produced by a current i in a long straight wire. The field $d\vec{B}$ at P associated with the current-length element $i\,d\vec{s}$ is directed into the page, as shown.

$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds \sin\theta}{r^2}.$$

$$B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi} \int_0^\infty \frac{\sin\theta\,ds}{r^2}.$$

$$r = \sqrt{s^2 + R^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}}.$$

$$B = \frac{\mu_0 i}{2\pi} \int_0^\infty \frac{R\,ds}{(s^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0 i}{2\pi R} \left[\frac{s}{(s^2 + R^2)^{1/2}} \right]_0^\infty = \frac{\mu_0 i}{2\pi R}$$

$$B = \frac{\mu_0 i}{4\pi R} \qquad \text{(semi-infinite straight wire).}$$

Magnetic Field due to a Current in a Circular Arc of Wire:

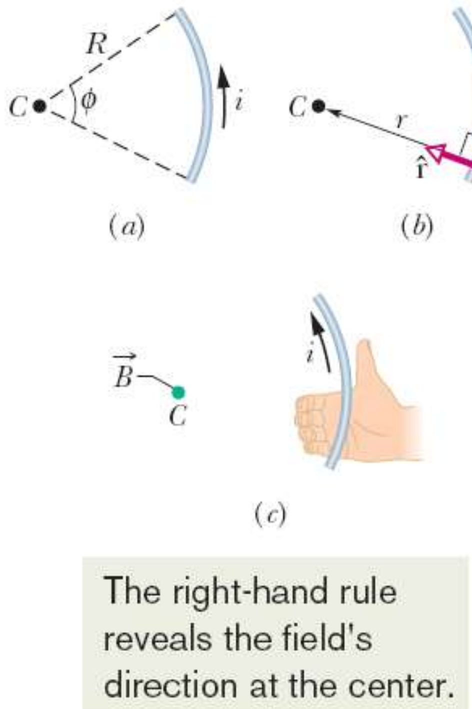


Fig. 29-6 (*a*) A wire in the shape of a circular arc with center C carries current i. (*b*) For any element of wire along the arc, the angle between the directions of $d\vec{s}$ and $\hat{r}$ is 90°. (*c*) Determining the direction of the magnetic field at the center C due to the current in the wire; the field is out of the page, in the direction of the fingertips, as indicated by the colored dot at C.

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds \sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2}.$$

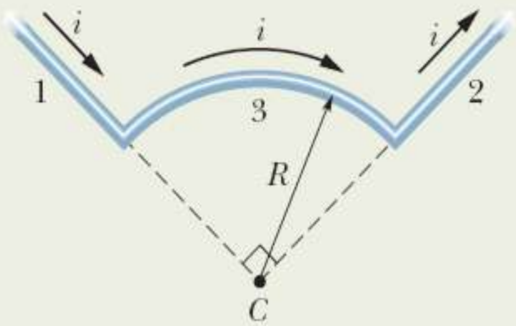
$$B = \int dB = \int_0^{\phi} \frac{\mu_0}{4\pi}\frac{iR\,d\phi}{R^2} = \frac{\mu_0 i}{4\pi R}\int_0^{\phi} d\phi.$$

$$\boxed{B = \frac{\mu_0 i \phi}{4\pi R} \qquad \text{(at center of circular arc).}}$$

$$B = \frac{\mu_0 i(2\pi)}{4\pi R} = \frac{\mu_0 i}{2R} \qquad \text{(at center of full circle).}$$

Example, Magnetic field at the center of a circular arc of a circle.:

The wire in Fig. 29-7*a* carries a current i and consists of a circular arc of radius R and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center C of the arc. What magnetic field $\vec{B}$ (magnitude and direction) does the current produce at C?



(*a*)

Straight sections: For any current-length element in section 1, the angle θ between $d\vec{s}$ and $\hat{r}$ is zero (Fig. 29-7*b*); so Eq. 29-1 gives us

$$dB_1 = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 0}{r^2} = 0.$$

Thus, the current along the entire length of straight section 1 contributes no magnetic field at C:

$$B_1 = 0.$$

The same situation prevails in straight section 2, where the angle θ between $d\vec{s}$ and $\hat{r}$ for any current-length element is 180°. Thus,

$$B_2 = 0.$$

Circular arc: Application of the Biot–Savart law to evaluate the magnetic field at the center of a circular arc leads to Eq. 29-9 ($B = \mu_0 i\phi/4\pi R$). Here the central angle ϕ of the arc is $\pi/2$ rad. Thus from Eq. 29-9, the magnitude of the magnetic field $\vec{B}_3$ at the arc's center C is

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}.$$

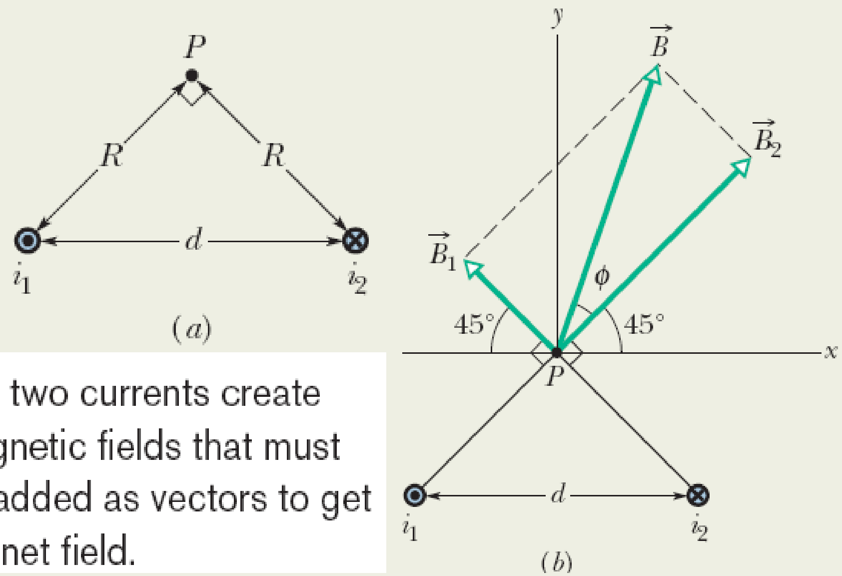
To find the direction of $\vec{B}_3$, we apply the right-hand rule displayed in Fig. 29-4. Mentally grasp the circular arc with your right hand as in Fig. 29-7*c*, with your thumb in the direction of the current. The direction in which your fingers curl around the wire indicates the direction of the magnetic field lines around the wire. They form circles around the wire, coming out of the page above the arc and going into the page inside the arc. In the region of point C (inside the arc), your fingertips point *into the plane* of the page. Thus, $\vec{B}_3$ is directed into that plane.

Net field: Generally, when we must combine two or more magnetic fields to find the net magnetic field, we must combine the fields as vectors and not simply add their magnitudes. Here, however, only the circular arc produces a magnetic field at point C. Thus, we can write the magnitude of the net field $\vec{B}$ as

$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}. \qquad \text{(Answer)}$$

Example, Magnetic field off to the side of two long straight currents:

Figure 29-8a shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point P? Assume the following values: $i_1 = 15$ A, $i_2 = 32$ A, and $d = 5.3$ cm.



The two currents create magnetic fields that must be added as vectors to get the net field.

Finding the vectors: In Fig. 29-8a, point P is distance R from both currents i_1 and i_2. Thus, Eq. 29-4 tells us that at point P those currents produce magnetic fields $\vec{B}_1$ and $\vec{B}_2$ with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi R}.$$

In the right triangle of Fig. 29-8a, note that the base angles (between sides R and d) are both 45°. This allows us to write $\cos 45° = R/d$ and replace R with $d \cos 45°$. Then the field magnitudes B_1 and B_2 become

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45°} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45°}.$$

Adding the vectors: We can now vectorially add $\vec{B}_1$ and $\vec{B}_2$ to find the net magnetic field $\vec{B}$ at point P, either by using a vector-capable calculator or by resolving the vectors into components and then combining the components of $\vec{B}$. However, in Fig. 29-8b, there is a third method: Because $\vec{B}_1$ and $\vec{B}_2$ are perpendicular to each other, they form the legs of a right triangle, with $\vec{B}$ as the hypotenuse. The Pythagorean theorem then gives us

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(\cos 45°)} \sqrt{i_1^2 + i_2^2}$$

$$= \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\sqrt{(15\ \text{A})^2 + (32\ \text{A})^2}}{(2\pi)(5.3 \times 10^{-2}\ \text{m})(\cos 45°)}$$

$$= 1.89 \times 10^{-4}\ \text{T} \approx 190\ \mu\text{T}. \qquad \text{(Answer)}$$

The angle ϕ between the directions of $\vec{B}$ and $\vec{B}_2$ in Fig. 29-8b follows from

$$\phi = \tan^{-1}\frac{B_1}{B_2},$$

which, with B_1 and B_2 as given above, yields

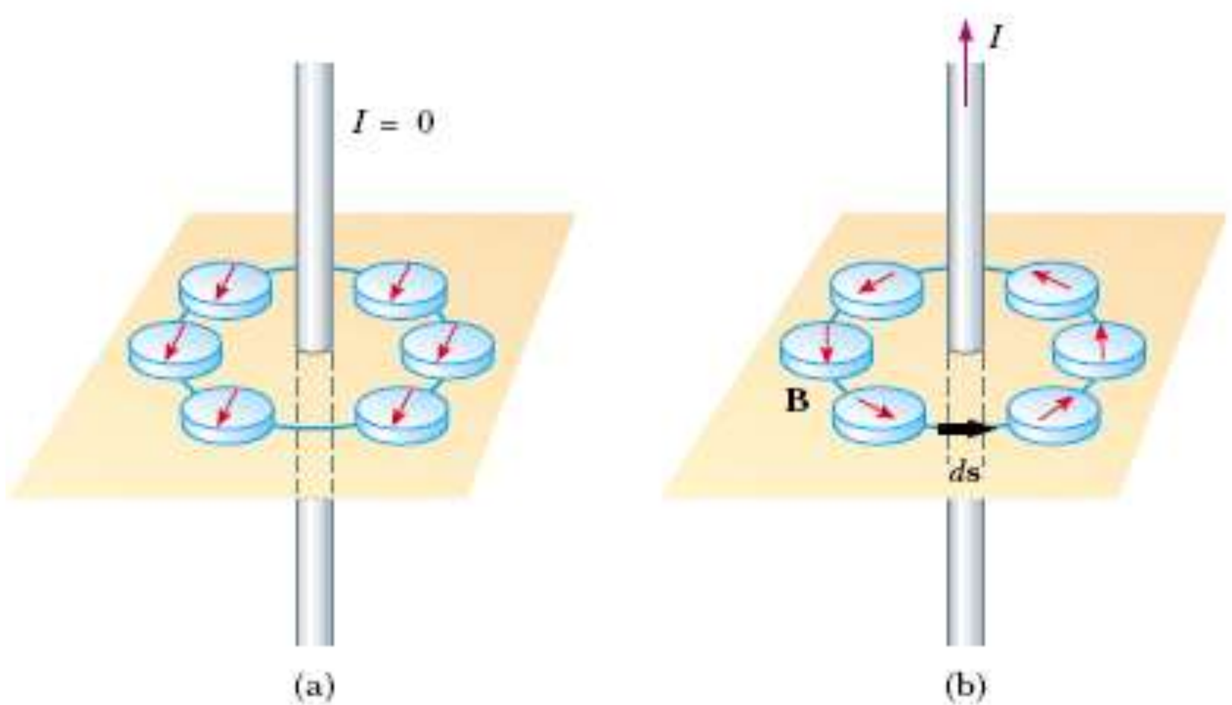
$$\phi = \tan^{-1}\frac{i_1}{i_2} = \tan^{-1}\frac{15\ \text{A}}{32\ \text{A}} = 25°.$$

The angle between the direction of $\vec{B}$ and the x axis shown in Fig. 29-8b is then

$$\phi + 45° = 25° + 45° = 70°. \qquad \text{(Answer)}$$

Ampere’s Law

- Several compasses are placed in a loop in a horizontal plane near a long vertical wire.
  - When there is no current in the wire, all compasses in the loop point in the same direction (the direction of the Earth’s magnetic field).
  - When the wire carries a strong steady current I, the compass needles will all deflect in a direction tangent to the circle.



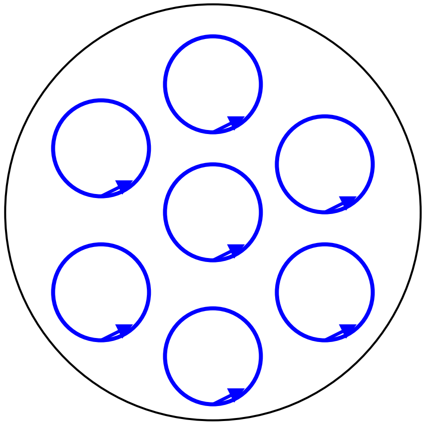
- The direction of the deflection is determined by the right hand rule: if the wire is grasped in the right hand with the thumb in the direction of the current I, the fingers curl in the direction of the magnetic field B produced by the current in the wire.
- When the current I is reversed, the direction of the deflection in the compasses will also reverse.
- The compass needles point in the direction of the magnetic field B, therefore, the lines of B form circles around the wire as previously discussed.

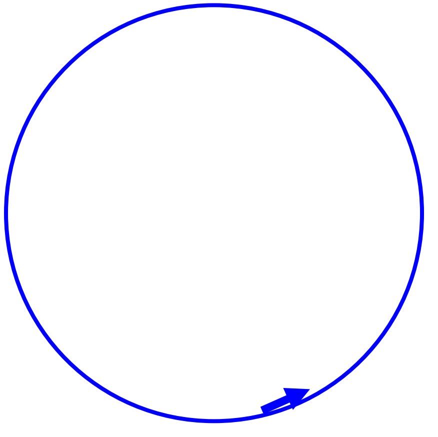
- The magnitude of B is the same everywhere on a circular path centered on the wire and lying in the plane that is perpendicular to the wire.
- The magnetic field B is directly proportional to the current and inversely proportional to the square of the distance from the wire (as described in the Biot-Savart law).
- For a circular path surrounding a wire, divide the circular path into small elements of length ds and evaluate the dot product B•ds over the entire circumference of the circle.

B field of large current loop

- Electrostatics – began with sheet of electric monopoles
- Magnetostatics – begin sheet of magnetic dipoles
- Sheet of magnetic dipoles equivalent to current loop
- Magnetic moment for one dipole $m = I\ \alpha$ area α

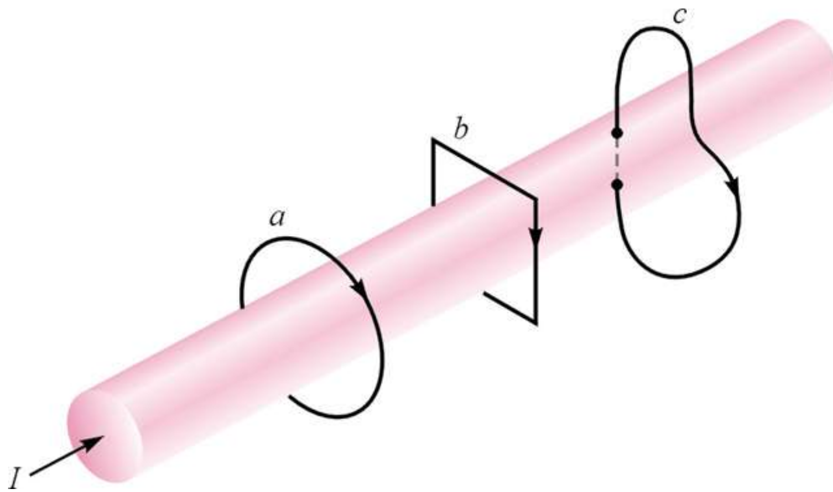
  for loop $M = I\ A$ area A





- Magnetic dipoles one current loop
- Evaluate B field along axis passing through loop

Ampere’s Circuital Law



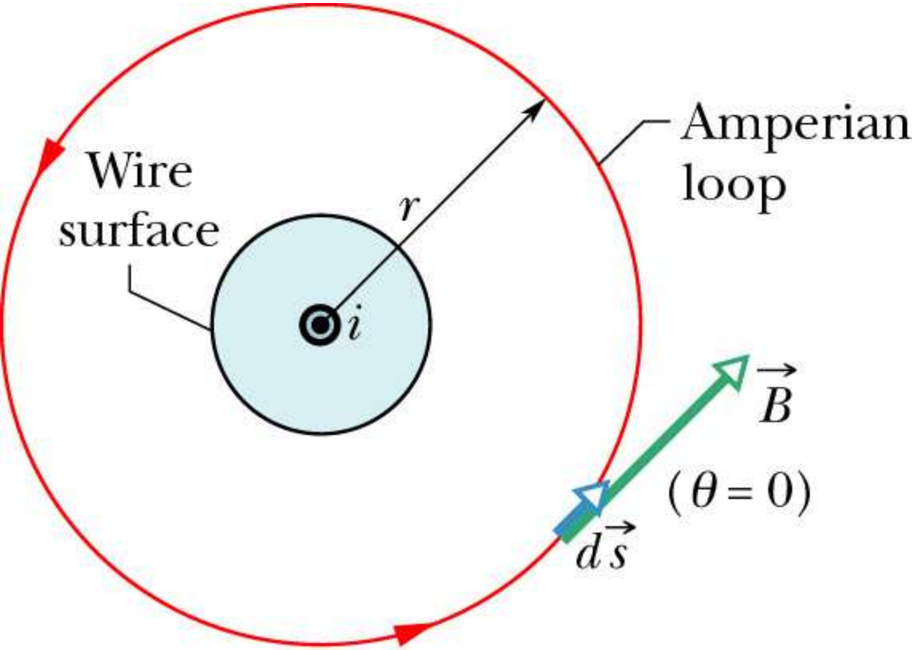
The magnetic field in space around an electric current is proportional to the electric current which serves as its source, just as the electric field in space is proportional to the charge which serves as its source.

- The vectors ds and B are parallel to each other at each point:

$$\vec{B} \bullet d\vec{s} = \vec{B} \cdot d\vec{s} \cdot \cos\theta$$

$$\vec{B} \bullet d\vec{s} = \vec{B} \cdot d\vec{s} \cdot \cos 0$$

$$\vec{B} \bullet d\vec{s} = \vec{B} \cdot d\vec{s}$$



$$\oint \vec{B} \bullet d\vec{s} = \vec{B} \cdot \oint d\vec{s}$$

- Integrate around the circumference of the circle. Pull B out in front of the integral because it is constant at every point on the

- The equation for the magnetic field B around a straight conductor is: $\vec{B} = \frac{\mu_o \cdot I}{2 \cdot \pi \cdot r}$

- The integral $\oint d\vec{s} = 2 \cdot \pi \cdot r$ is the circumference of the circle.

- Ampere's law:

$$\oint \vec{B} \bullet d\vec{s} = \frac{\mu_o \cdot I}{2 \cdot \pi \cdot r} \cdot 2 \cdot \pi \cdot r$$

$$\oint \vec{B} \bullet d\vec{s} = \mu_o \cdot I$$

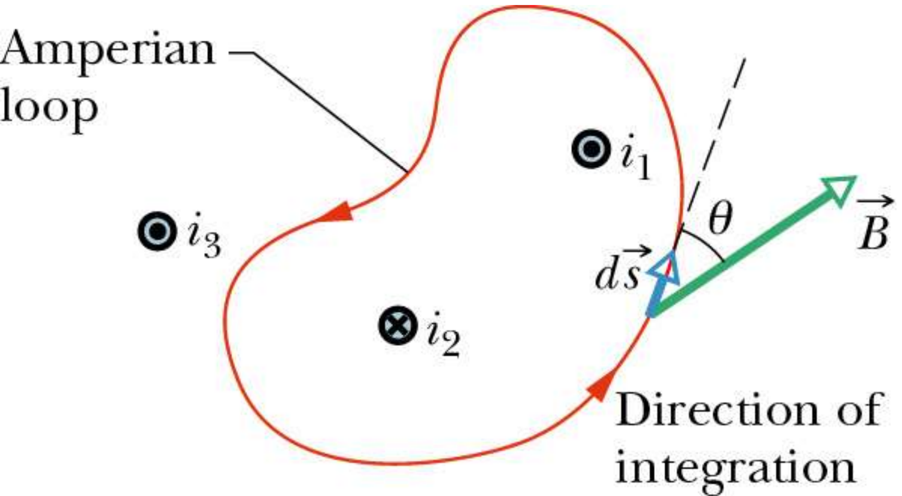
- Ampere’s law applies to any closed path surrounding a steady current.
- Ampere’s law states that the line integral B•ds around any closed path equals $\mu_o \cdot I$, where I is the total steady current passing through any surface bounded by the closed path.
- Ampere’s law:

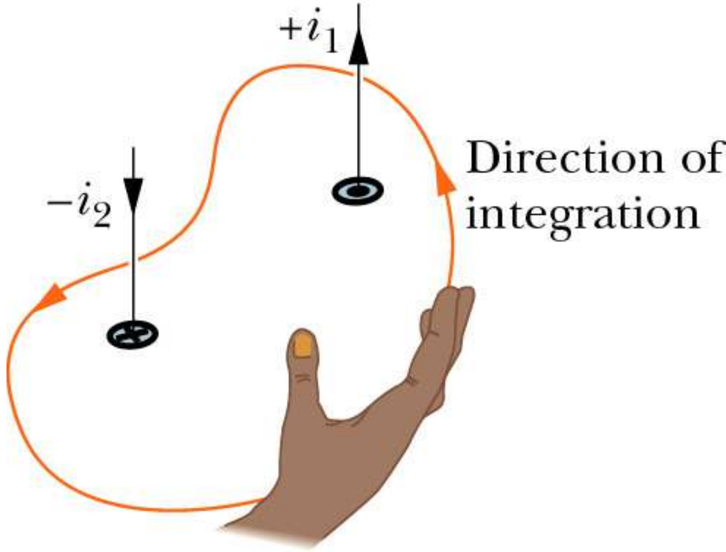
$$\oint \vec{B} \bullet d\vec{s} = \mu_o \cdot I_{enclosed}$$

- Ampere’s law only applies to steady currents.

- Ampere’s Law is used for calculating the magnetic field of current configurations with a high degree of symmetry just like Gauss’ law is used to calculate the electric field of highly symmetric charge distributions.
- The line that is drawn around the conductors to determine the magnetic field is called an Amperian loop.
- The direction of the magnetic field B is assumed to be in the direction of integration.
- Use the right hand rule to assign a plus or minus sign to each of the currents that make up the net enclosed current $I_{enclosed}$.

- Curl the fingers of the right hand around the Amperian loop in the direction of integration.
  - A current passing through the loop in the direction of the thumb is assigned a plus sign.
  - A current passing through the loop in the opposite direction of the thumb is assigned a minus sign.
- If B is positive, the direction we assumed for B is correct; if B is negative, neglect the minus sign and redraw B in the opposite direction.





$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law).}$$

Only the currents encircled by the loop are used in Ampere's law.

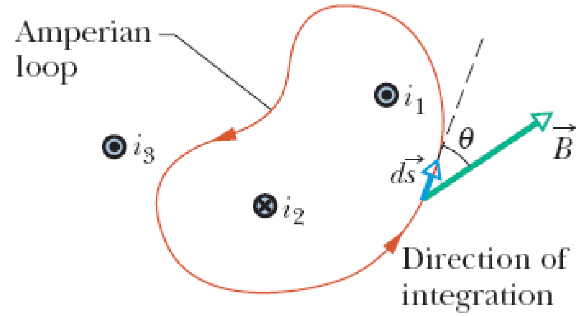


Fig. 29-11 Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

This is how to assign a sign to a current used in Ampere's law.

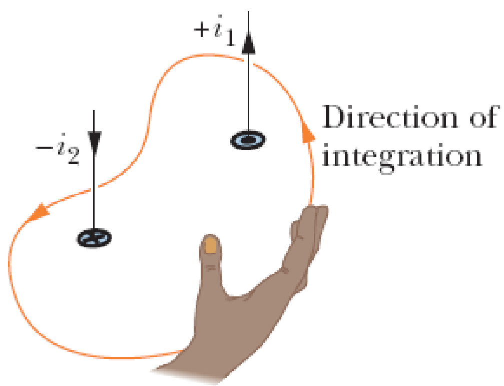


Fig. 29-12 A right-hand rule for Ampere's law, to determine the signs for currents encircled by an Amperian loop. The situation is that of Fig. 29-11.

Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

29.4: Ampere's Law, Magnetic Field Outside a Long Straight Wire Carrying Current:

All of the current is encircled and thus all is used in Ampere's law.

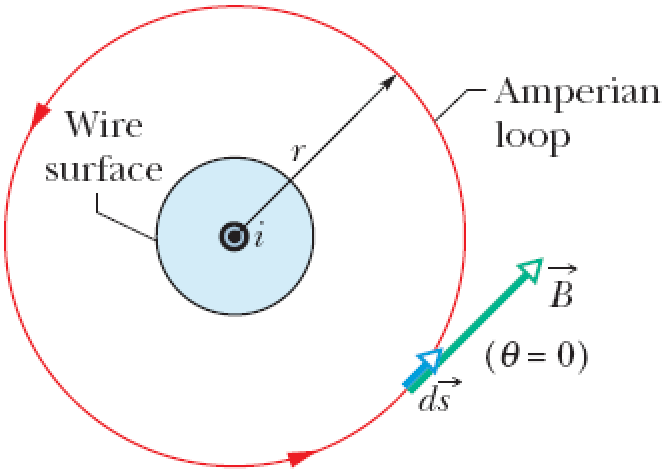


Fig. 29-13 Using Ampere's law to find the magnetic field that a current i produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = B \oint ds = B(2\pi r).$$

$$B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r} \qquad \text{(outside straight wire).}$$

29.4: Ampere's Law, Magnetic Field Inside a Long Straight Wire Carrying Current:

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r).$$

$$i_{\text{enc}} = i\,\frac{\pi r^2}{\pi R^2}.$$

$$B(2\pi r) = \mu_0 i\,\frac{\pi r^2}{\pi R^2}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r \qquad \text{(inside straight wire).}$$

Only the current encircled by the loop is used in Ampere's law.

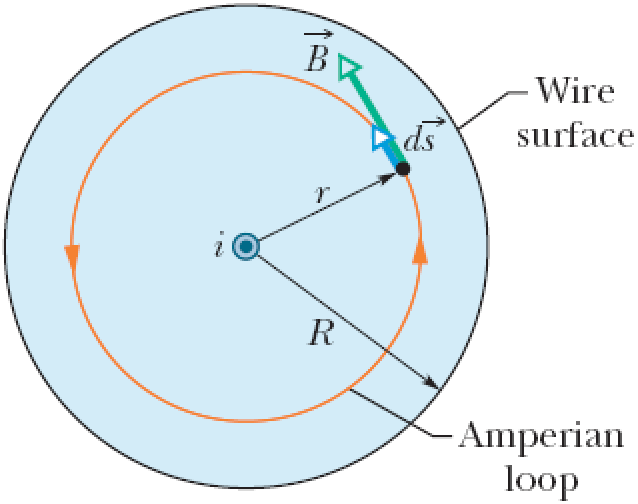
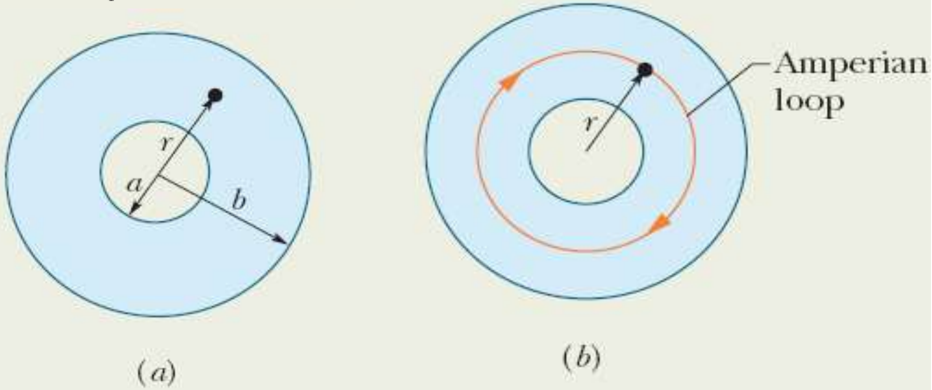


Fig. 29-14 Using Ampere's law to find the magnetic field that a current i produces inside a long straight wire of circular cross section. The current is uniformly distributed over the cross section of the wire and emerges from the page. An Amperian loop is drawn inside the wire.

Example, Ampere's Law to find the magnetic field inside a long cylinder of current.

Figure 29-15*a* shows the cross section of a long conducting cylinder with inner radius $a = 2.0$ cm and outer radius $b = 4.0$ cm. The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by $J = cr^2$, with $c = 3.0 \times 10^6$ A/m^4 and r in meters. What is the magnetic field $\vec{B}$ at the dot in Fig. 29-15*a*, which is at radius $r = 3.0$ cm from the central axis of the cylinder?



Calculations: We write the integral as

$$i_{enc} = \int J\,dA = \int_a^r cr^2(2\pi r\,dr)$$

$$= 2\pi c \int_a^r r^3\,dr = 2\pi c\left[\frac{r^4}{4}\right]_a^r$$

$$= \frac{\pi c(r^4 - a^4)}{2}.$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc},$$

gives us

$$B(2\pi r) = -\frac{\mu_0 \pi c}{2}(r^4 - a^4).$$

Solving for B and substituting known data yield

$$B = -\frac{\mu_0 c}{4r}(r^4 - a^4)$$

$$= -\frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(3.0 \times 10^6\,\text{A/m}^4)}{4(0.030\,\text{m})}$$

$$\times [(0.030\,\text{m})^4 - (0.020\,\text{m})^4]$$

$$= -2.0 \times 10^{-5}\,\text{T}.$$

Thus, the magnetic field $\vec{B}$ at a point 3.0 cm from the central axis has magnitude

$$B = 2.0 \times 10^{-5}\,\text{T} \qquad \text{(Answer)}$$

Solenoids and Toroids:

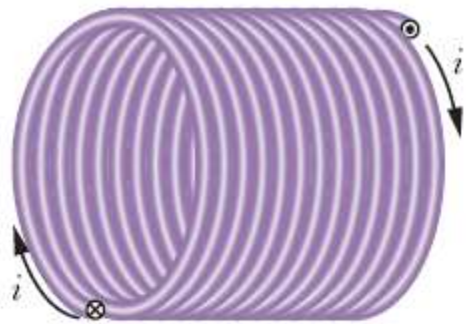


Fig. 29-16 A solenoid carrying current i.

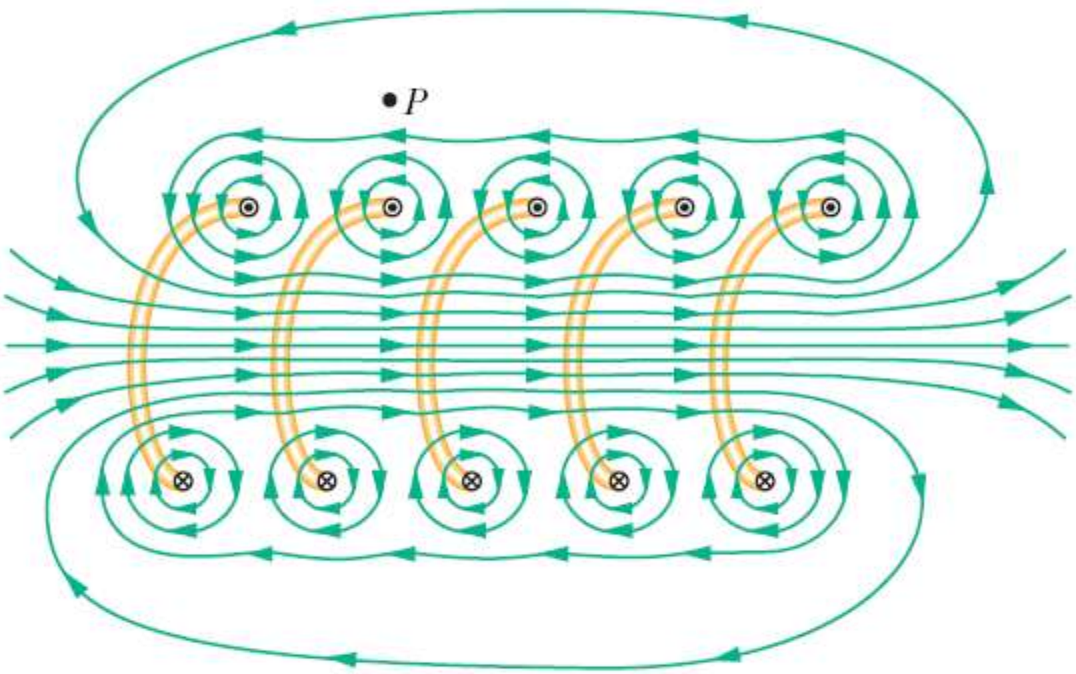
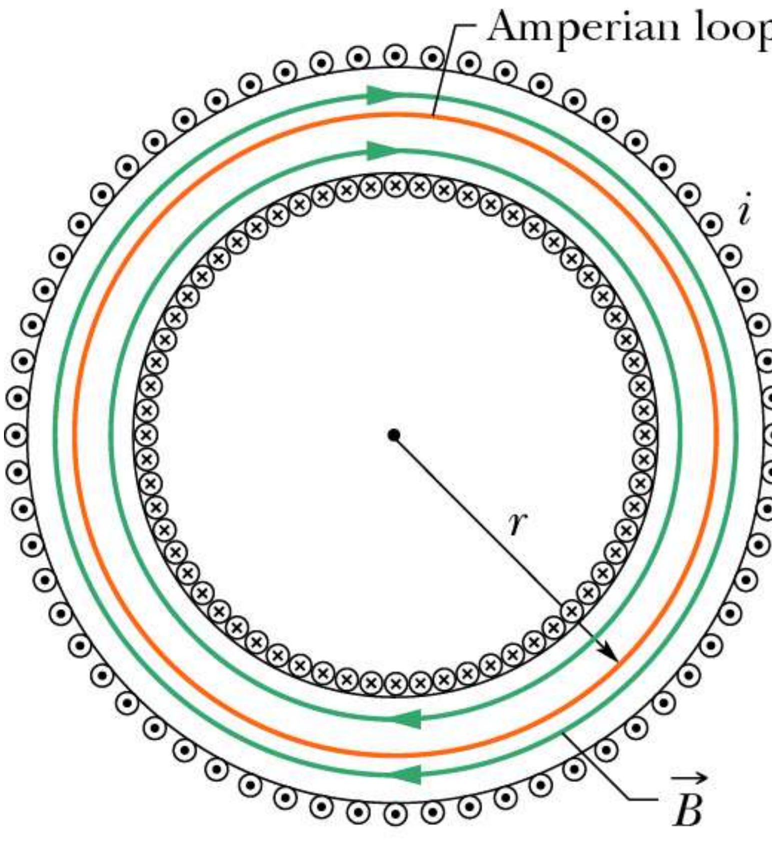


Fig. 29-17 A vertical cross section through the central axis of a "stretched-out" solenoid. The back portions of five turns are shown, as are the magnetic field lines due to a current through the solenoid. Each turn produces circular magnetic field lines near itself. Near the solenoid's axis, the field lines combine into a net magnetic field that is directed along the axis. The closely spaced field lines there indicate a strong magnetic field. Outside the solenoid the field lines are widely spaced; the field there is very weak.

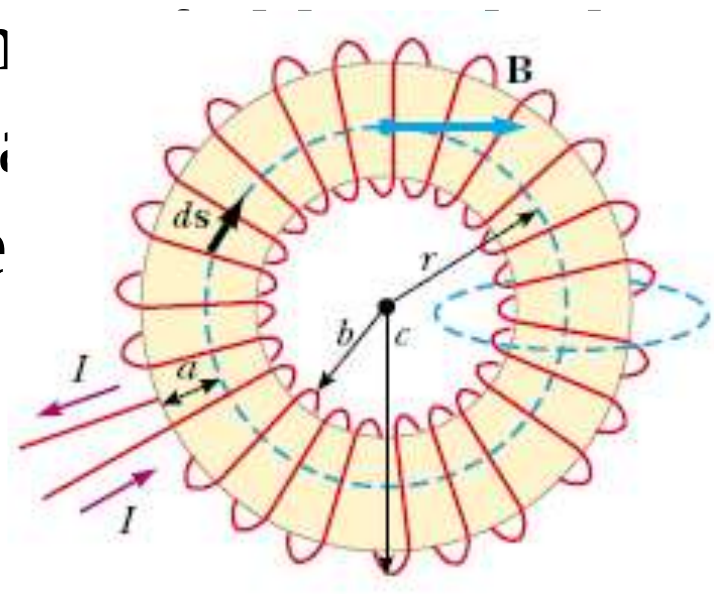
The Magnetic Field of a Toroidal Coil

- A toroidal coil consists of N turns of wire wrapped around a donut-shaped structure. Assuming that the turns are closely spaced, determine the magn
coil,
cente

(a)



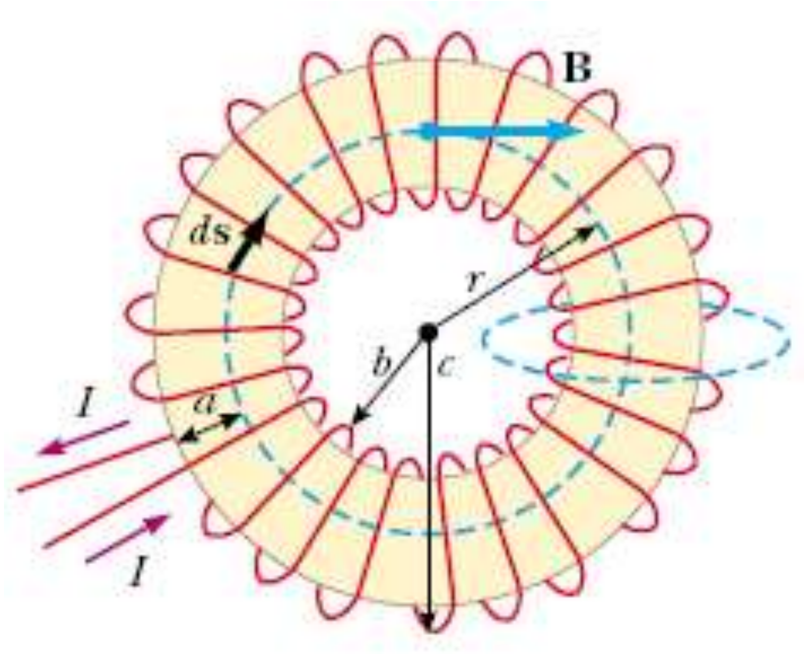
(b)



- To determine the magnetic field B inside the coil, evaluate the line integral B•ds over an Amperian loop of radius r.
- The magnetic field B is constant along the Amperian loop of radius r and tangent to the loop at every point on the loop.
- For N loops:

$$\oint \vec{B} \bullet d\vec{s} = N \cdot \mu_o \cdot I_{enclosed}$$

$$\vec{B} \cdot \oint d\vec{s} = N \cdot \mu_o \cdot I_{enclosed}$$



- The integral of ds over the closed Amperian loop:

$$\oint d\vec{s} = s \qquad s = 2 \cdot \pi \cdot r$$

- Solving for the magnetic field B:

$$\vec{B} \cdot 2 \cdot \pi \cdot r = N \cdot \mu_o \cdot I_{enclosed}$$

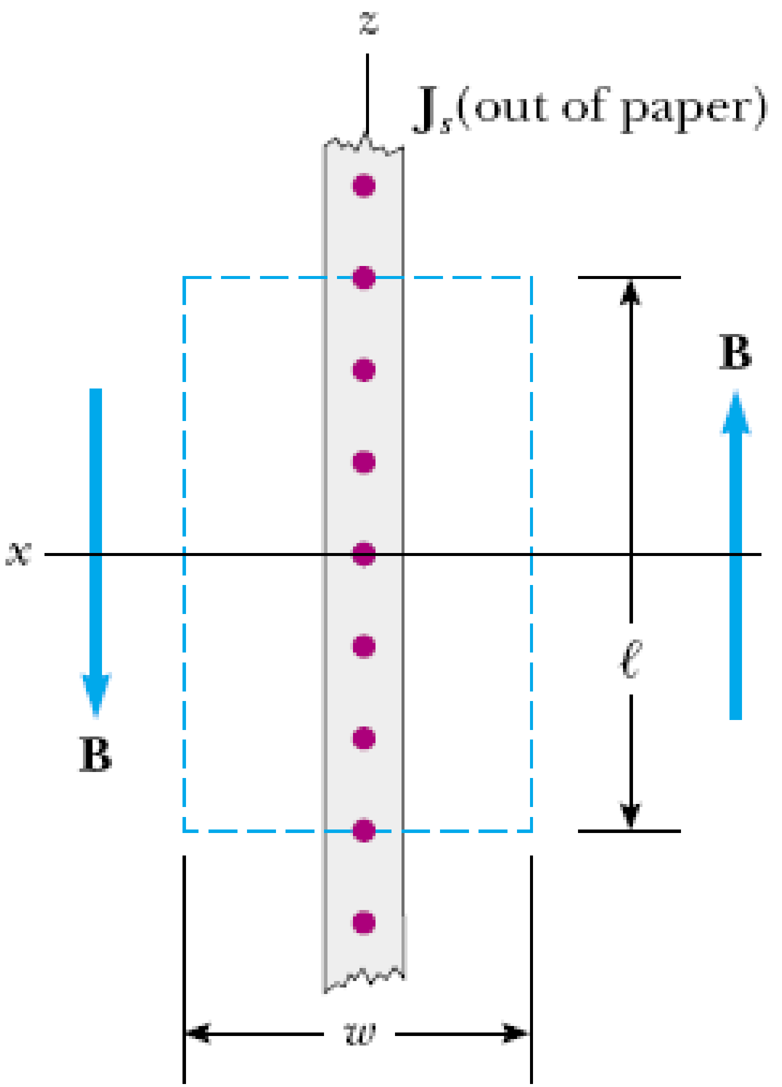
$$\vec{B} = \frac{N \cdot \mu_o \cdot I_{enclosed}}{2 \cdot \pi \cdot r}$$

- Within the toroidal coil, the magnetic field B varies as 1/r, therefore, the magnetic field B is not uniform within the coil.
- If r is large compared with **a**, where **a** is the cross-sectional radius of the toroid, the magnetic field will be approximately uniform inside the coil.
- For an “ideal” toroidal coil in which the turns are closely spaced, the magnetic field outside the coil is zero (0 T).
  - The net current enclosed by an Amperian loop located outside the toroidal coil is 0 A.

- Ampere’s law returns a value of 0 T for the magnetic field outside the toroidal coil.
- The turns of a toroidal coil actually form a helix rather than circular loops, so there is always a small magnetic field found outside the toroidal coil.
- For the donut hole, the enclosed current for an Amperian loop within the hole area is 0 A, therefore, the value for the magnetic field inside the donut hole is 0 T.

Magnetic Field of an Infinite Current Sheet

- An infinite sheet lying in the yz plane carries a surface current of density Js.
  - The current is in the y direction.
  - Js represents the current per unit length measured along the z axis.



- To determine the magnetic field B near the sheet, draw an Amperian rectangle through the sheet.
  - The rectangle has dimensions l and w, where the sides of length l are parallel to the surface of the sheet.
  - The net current through the Amperian rectangle is Js·l (the current per unit length times the length of the rectangle).
- Apply Ampere's law:

$$\oint \vec{B} \bullet d\vec{s} = \mu_o \cdot I_{enclosed}$$

$$\vec{B} \cdot \oint d\vec{s} = \mu_o \cdot I_{enclosed}$$

- The integral of ds over the closed Amperian loop should be $s = 2\cdot l + 2\cdot w$, however, there is no component of the magnetic field in the direction of the sides w.
- The integral of ds over the Amperian loop is $s = 2\cdot l$.
- $I_{enclosed} = Js\cdot l$.
- Combining the equations:

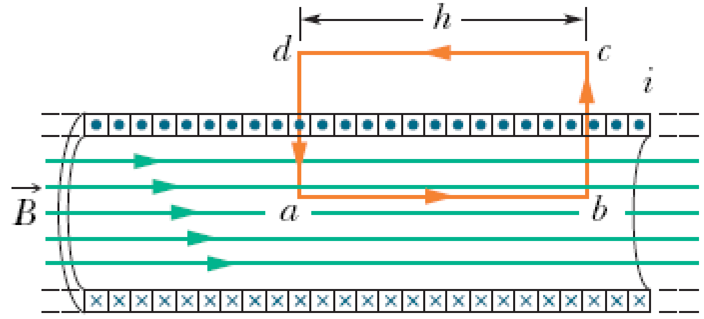
$$\vec{B}\cdot 2\cdot l = \mu_o \cdot J_s \cdot l$$

$$\vec{B} = \frac{\mu_o \cdot J_s \cdot l}{2\cdot l} \qquad \vec{B} = \frac{\mu_o \cdot J_s}{2}$$

- The magnetic field is independent of the distance from the current sheet.
- The magnetic field is uniform and is parallel to the plane of the sheet.

Solenoids:

Fig. 29-19 Application of Ampere's law to a section of a long ideal solenoid carrying a current *i*. The Amperian loop is the rectangle *abcda*.



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}},$$

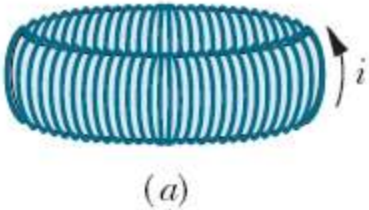
$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}.$$

$i_{\text{enc}} = i(nh).$ Here *n* be the number of turns per unit length of the solenoid

$$Bh = \mu_0 inh$$

$$B = \mu_0 in \qquad \text{(ideal solenoid).}$$

Magnetic Field of a Toroid:



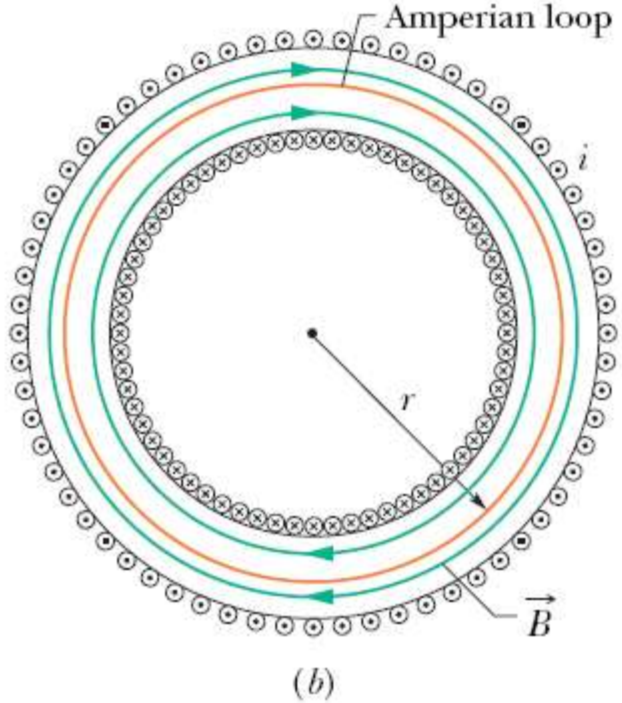


Fig. 29-20 (*a*) A toroid carrying a current *i*. (*b*) A horizontal cross section of the toroid. The interior magnetic field (inside the bracelet-shaped tube) can be found by applying Ampere's law with the Amperian loop shown.

$$(B)(2\pi r) = \mu_0 i N,$$

where *i* is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and *N* is the total number of turns. This gives

$$B = \frac{\mu_0 i N}{2\pi} \frac{1}{r} \qquad \text{(toroid).}$$

Example, The field inside a solenoid:

A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length L. What is B at its center?

KEY IDEA

The magnitude B of the magnetic field along the solenoid's central axis is related to the solenoid's current i and number of turns per unit length n by Eq. 29-23 ($B = \mu_0 in$).

Calculation: Because B does not depend on the diameter of the windings, the value of n for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

$$B = \mu_0 in = (4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(5.57\ \mathrm{A}) \frac{5 \times 850\ \text{turns}}{1.23\ \mathrm{m}}$$

$$= 2.42 \times 10^{-2}\ \mathrm{T} = 24.2\ \mathrm{mT}. \qquad \text{(Answer)}$$

To a good approximation, this is the field magnitude throughout most of the solenoid.

A Current Carrying Coil as a Magnetic Dipole:

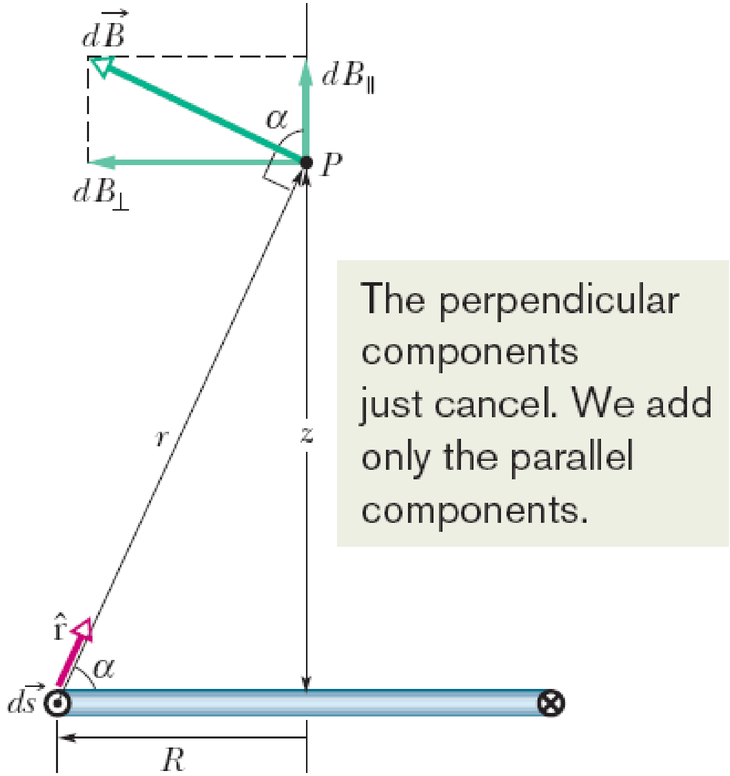


Fig. 29-22 Cross section through a current loop of radius R. The plane of the loop is perpendicular to the page, and only the back half of the loop is shown. We use the law of Biot and Savart to find the magnetic field at point P on the central perpendicular axis of the loop.

$$dB = \frac{\mu_0}{4\pi} \frac{i\, ds \sin 90°}{r^2}.$$

$$dB_{\parallel} = dB \cos \alpha. = \frac{\mu_0 i \cos \alpha\, ds}{4\pi r^2}.$$

$$r = \sqrt{R^2 + z^2}$$

$$\cos \alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}}.$$

$$dB_{\parallel} = \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}}\, ds.$$

$$B = \int dB_{\parallel}$$

$$= \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} \int ds$$

$$\boxed{B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}.}$$

A Current Carrying Coil as a Magnetic Dipole:

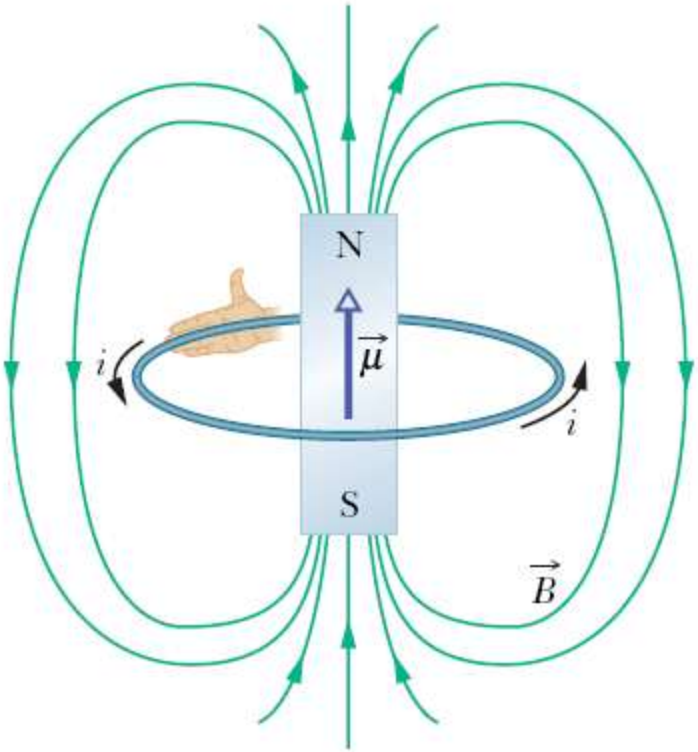


Fig. 29-21 A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles. The magnetic dipole moment $\vec{\mu}$ of the loop, its direction given by a curled–straight right-hand rule, points from the south pole to the north pole, in the direction of the field $\vec{B}$ within the loop.

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}},$$

$$z \gg R$$

$$B(z) \approx \frac{\mu_0 i R^2}{2z^3}.$$

$$B(z) = \frac{\mu_0}{2\pi}\frac{NiA}{z^3}.$$

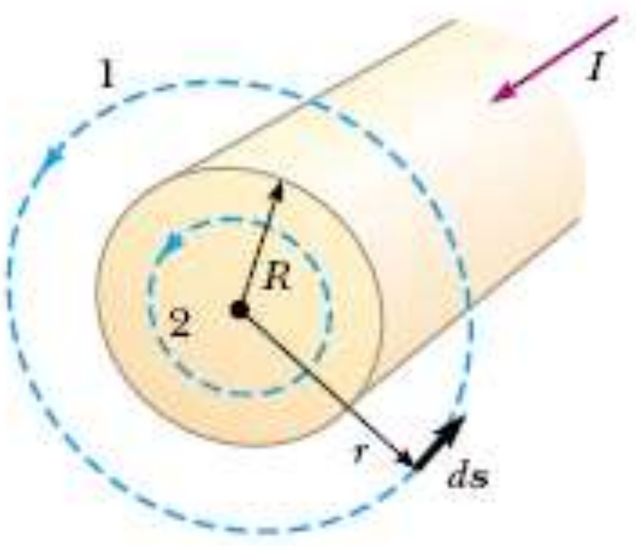
$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3} \qquad \text{(current-carrying coil).}$$

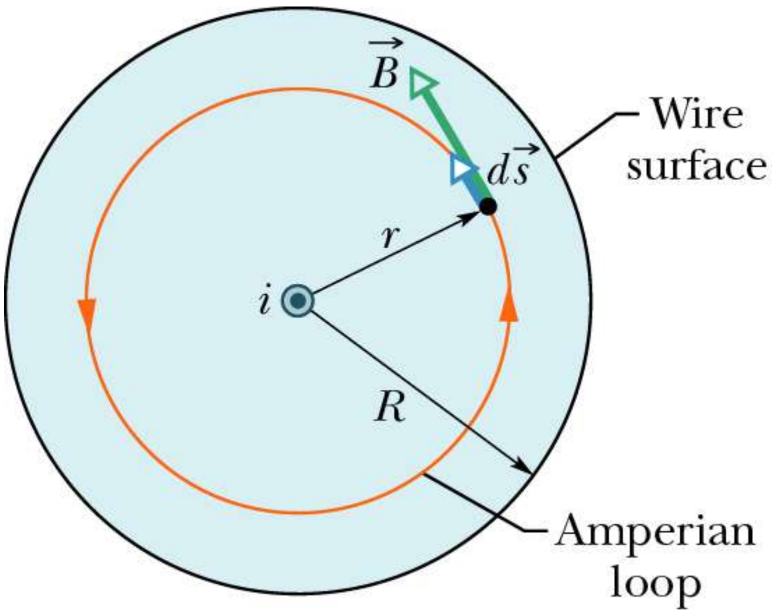
A general form for the magnetic dipole field is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{3\hat{r}(\vec{m}\cdot\hat{r}) - \vec{m}}{r^3}$$

Magnetic Field of a Long Wire

- A long, straight wire of radius R carries a steady current I_o that is uniformly distributed through the cross section of the wire.
- To determine the magnetic field at a distance r from the center of the wire to a point less than or equal to R ($r < R$):





- Draw a circular path (an Amperian loop) of radius r centered along the axis of the wire.
- Based on the symmetry of the circle, B must be constant in magnitude and parallel to ds at every point on the path of radius r.
- The total current passing through the Amperian loop is not I_o; the total current is less than I_o.
- The current is uniformly distributed throughout the cross-section of the wire; the enclosed current is proportional to the area of the Amperian loop.

- Proportional relationship between current and area:

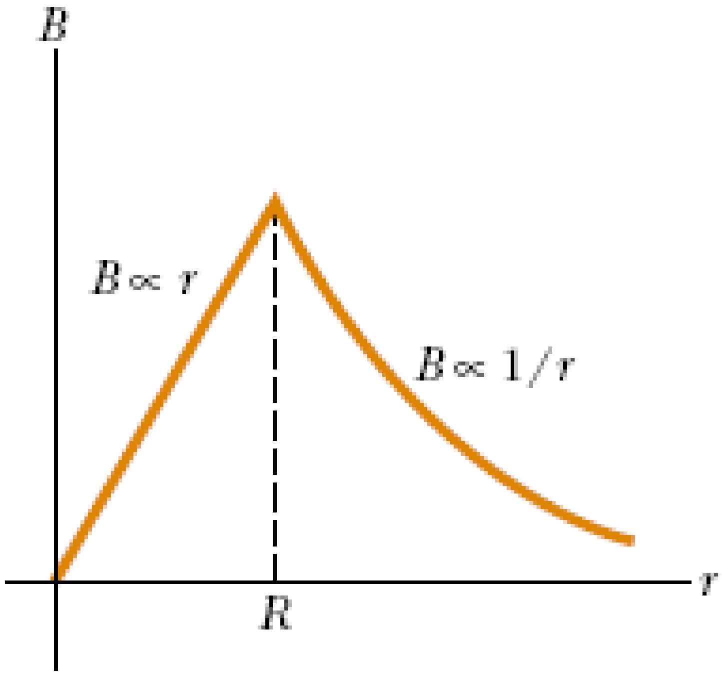
$$\frac{I_o}{\pi \cdot R^2} = \frac{I_{enclosed}}{\pi \cdot r^2} \qquad I_{enclosed} = \frac{I_o \cdot \pi \cdot r^2}{\pi \cdot R^2}$$

$$I_{enclosed} = \frac{I_o \cdot r^2}{R^2}$$

- Applying Ampere's law:

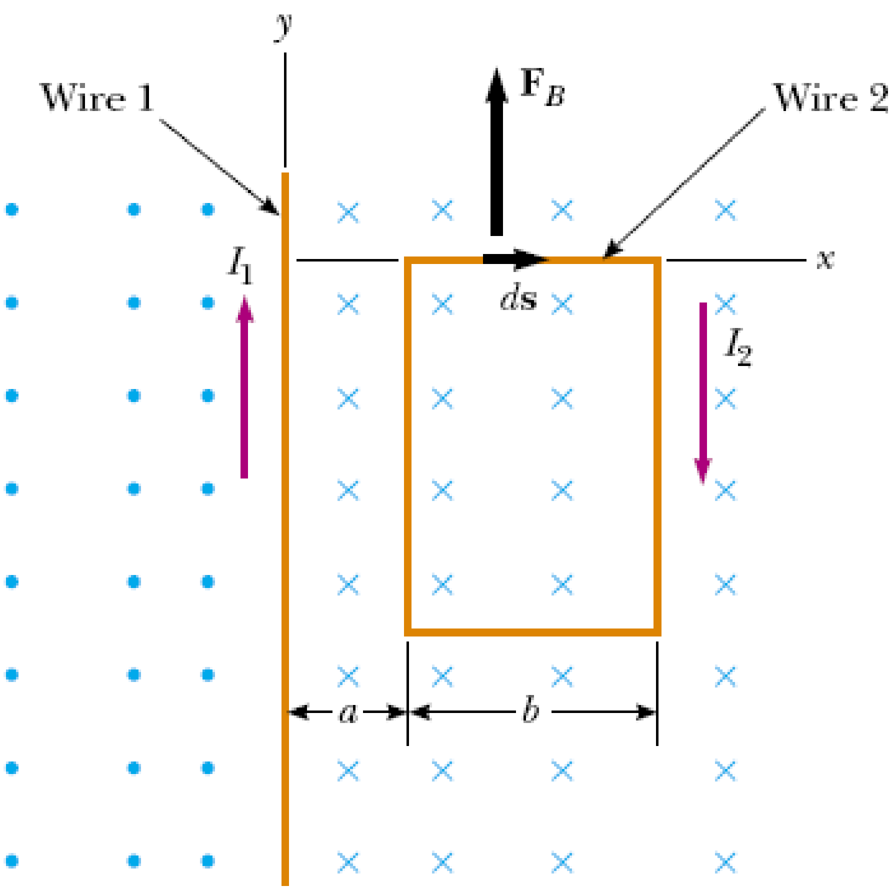
$$\oint \vec{B} \bullet d\vec{s} = \mu_o \cdot I_{enclosed}$$

$$\oint \vec{B} \bullet d\vec{s} = \frac{\mu_o \cdot I_o \cdot r^2}{R^2}$$



- The magnetic field B versus r is shown in the figure.
- Inside the wire, B $\rightarrow$ 0 as r $\rightarrow$ 0 and the strength of the magnetic field increases asbr $\rightarrow$ R.
- Outside the wire, the magnetic field B is proportional to 1/r.

Magnetic Force on a Current Segment



- A long straight wire along the y axis carries a steady current I_1.
- A rectangular circuit carries a current I_2.
- To determine the magnetic force F_B on the upper horizontal section of the rectangle, start with the force on a s

$$d\vec{F} = I \cdot \left(d\vec{s} \times \vec{B} \right)$$

he conductor given by:

- The magnetic field B is the magnetic field due to the long straight wire at the position of the element of length ds.
  - This value will change with increasing distance from the wire carrying current I_1.
  - The equation for the field B at a distance x from the straight wire is:

$$\vec{B} = \frac{\mu_o \cdot I_1}{2 \cdot \pi \cdot x}$$

  - The direction of the field is into the page.
- The current in I·(ds x B) is I_2.

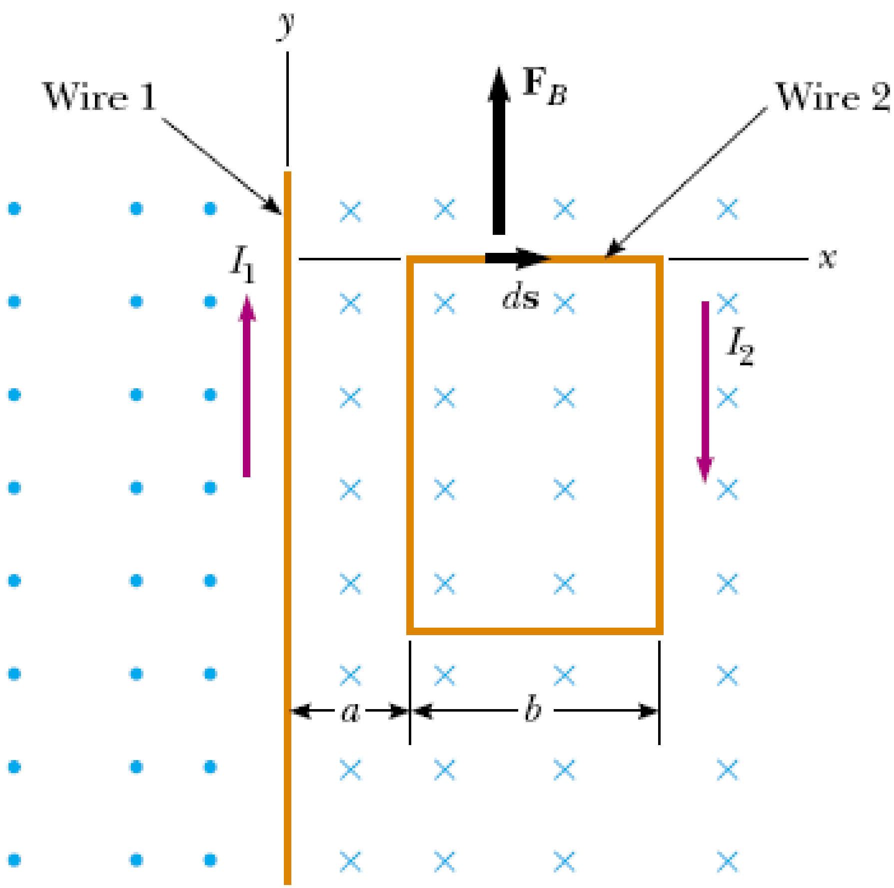
- Replace ds with dx because the value of x (the distance from the wire carrying current I_1 to the element of length ds) will change as we add up the elements of length ds along the x axis.
- The angle θ between dx and B is 90°.

$$d\vec{F} = I_2 \cdot dx \cdot \vec{B} \cdot \sin\theta$$

$$d\vec{F} = I_2 \cdot dx \cdot \vec{B} = I_2 \cdot dx \cdot \frac{\mu_o \cdot I_1}{2 \cdot \pi \cdot x}$$

$$d\vec{F} = \frac{\mu_o \cdot I_1 \cdot I_2}{2 \cdot \pi \cdot x} \cdot dx$$

- To determine the total force on the upper horizontal segment of the rectangular loop, integrate from length a to length a + b.



$$\int_a^{a+b} d\vec{F} = \int_a^{a+b} \frac{\mu_o \cdot I_1 \cdot I_2}{2 \cdot \pi \cdot x} \cdot dx$$

$$\vec{F} = \frac{\mu_o \cdot I_1 \cdot I_2}{2 \cdot \pi} \cdot \int_a^{a+b} \frac{1}{x} \cdot dx$$

$$\vec{F} = \frac{\mu_o \cdot I_1 \cdot I_2}{2 \cdot \pi} \cdot \left[\ln x\right]_a^{a+b}$$

$$\vec{F} = \frac{\mu_o \cdot I_1 \cdot I_2}{2 \cdot \pi} \cdot \left[\ln\left(a+b\right) - \ln a\right]$$

$$\vec{F} = \frac{\mu_o \cdot I_1 \cdot I_2}{2 \cdot \pi} \cdot \left[\ln \frac{a+b}{a}\right]$$

- The direction of the force is given by the right hand rule: fingers of right hand in direction of current I_2; palm facing in the direction of the magnetic field B; thumb points in the direction of the magnetic force F_B.
- The direction of the force is up toward the top of the page (board).
- The force on the bottom horizontal segment of the rectangular loop is equal in magnitude and opposite in direction to the force on the top horizontal segment of the rectangular loop.
- The forces on the sides of the rectangular loop are determined using the equations for parallel wires.

Magnetic vector potential

For an electrostatic field $\oint \mathbf{E}.\mathrm{d}\ell = 0 \qquad \mathbf{E} = -\nabla\phi$

$$\nabla \times \mathbf{E} = -\nabla \times \nabla\phi = 0$$

We cannot therefore represent **B** by e.g. the gradient of a scalar since

$$\nabla \times \mathbf{B} = \mu_o \mathbf{j} \quad (\textit{rhs} \text{ not zero})$$

$$\text{also } \nabla.\mathbf{B} = 0 \text{ always } (\nabla.\mathbf{E} = \frac{\rho}{\varepsilon_o})$$

Magnetostatic field, try

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla.\mathbf{B} = \nabla.(\nabla \times \mathbf{A}) = 0$$

$$\nabla \times \mathbf{B} = \nabla \times (\nabla \times \mathbf{A}) \text{(see later)}$$

B is unchanged by

$$\mathbf{A}' \rightarrow \mathbf{A} + \nabla\chi$$

$$\nabla \times \mathbf{A}' = \nabla \times (\mathbf{A} + \nabla\chi) = \nabla \times \mathbf{A} + 0$$

END