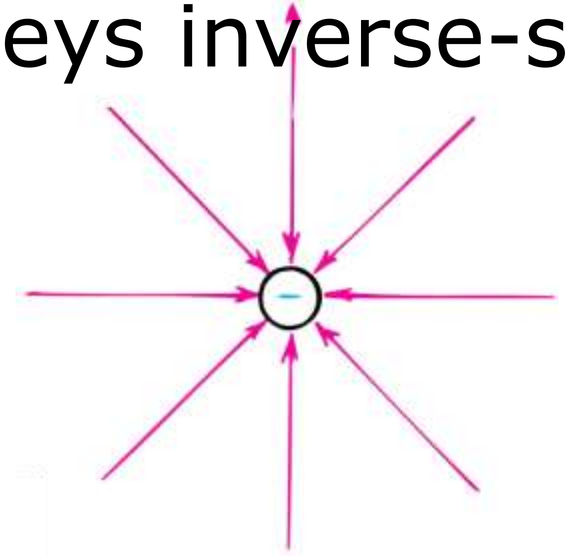
Electric Field

Electric field

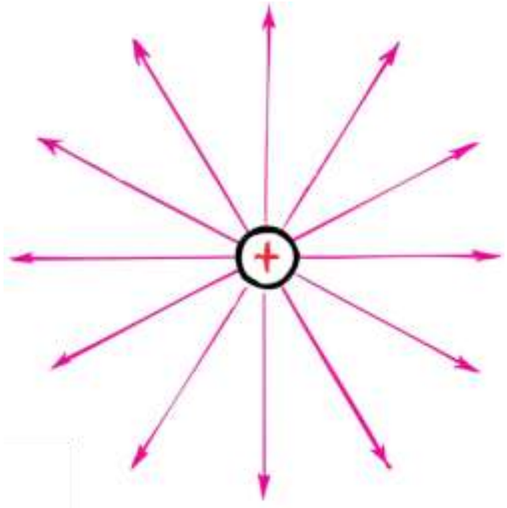
- Space surrounding an electric charge (an energetic aura)
- Describes electric force
- Around a charged particle obeys inverse-square law
- Force per unit charge



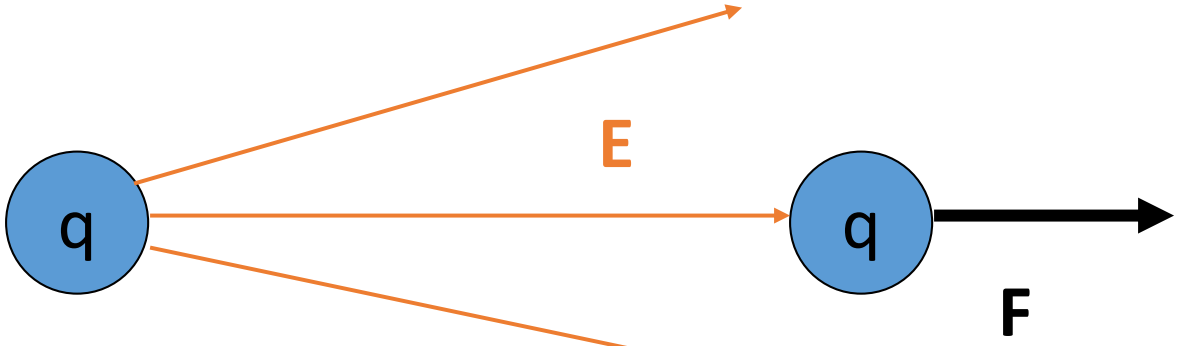
Electric Field

Electric field direction

- Same direction as the force on a positive charge
- Opposite direction to the force on an electron



The Field Formulation



ELECTRIC FIELD

- Qualitatively
  - The region of space around a charge where it can exert a force of electrical origin on another charge.
- Quantitatively
  - The intensity of ELECTRIC FIELD at any point is defined as the force exerted per unit charge by a positive test charge kept at that point.

$$E = \lim_{q_o \to 0} \left(\frac{F}{q_0} \right)$$

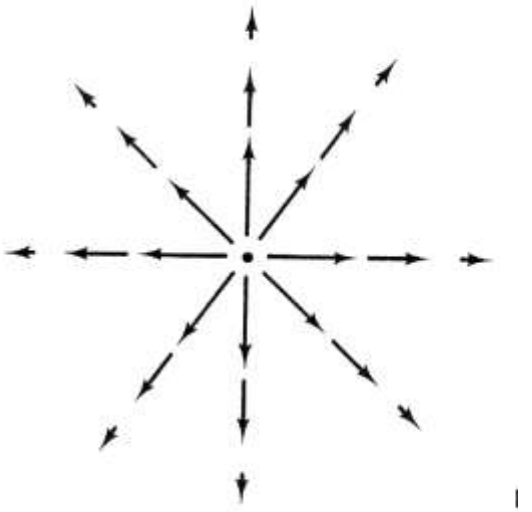
2.2.1 Fields lines and Gauss's law

A single point charge q, situated at the origin

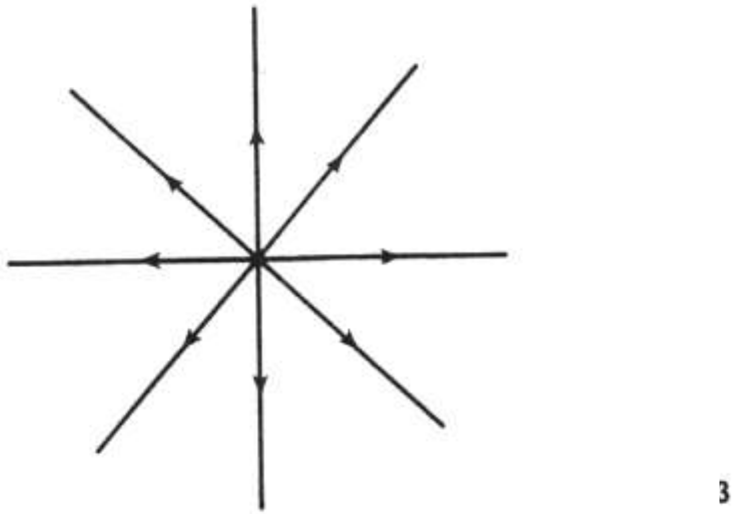
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Because the field falls off like $1/r^2$,the vectors get shorter as I go father away from the origin,and they always point radially outward.

This vectors can be connect up the arrows to form the *field lines.* The magnitude of the field is indicated by the density of the lines.



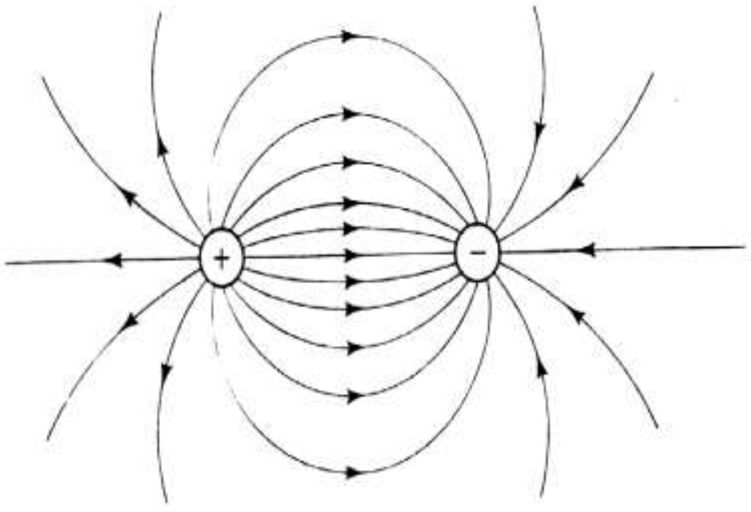
strength length of arrow



strength density of field line

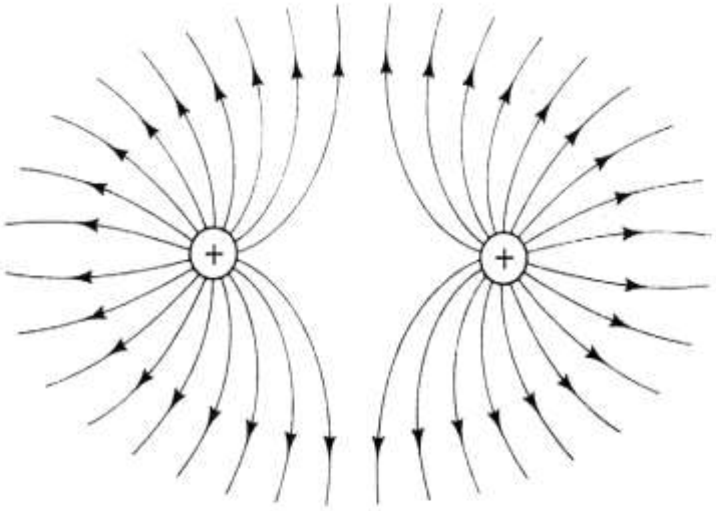
2.2.1 (2)

1. Field lines emanate from a point charge symmetrically in all directions.
2. Field lines originate on positive charges and terminate on negative ones.
3. They cannot simply stop in midair, though they may extend out to infinity.
4. Field lines can never cross.



Equal but opposite charges

Figure 2.14



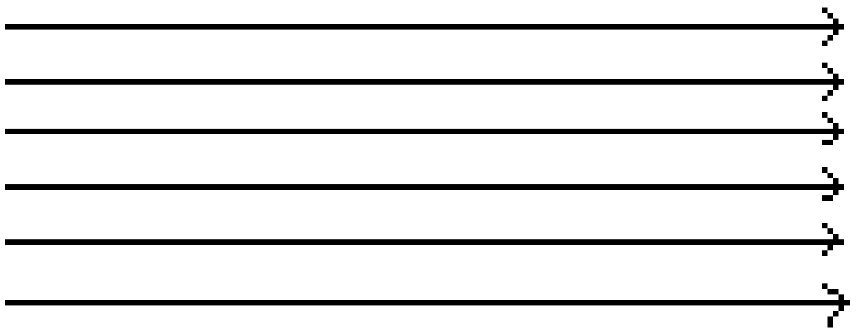
Equal charges

Figure 2.15

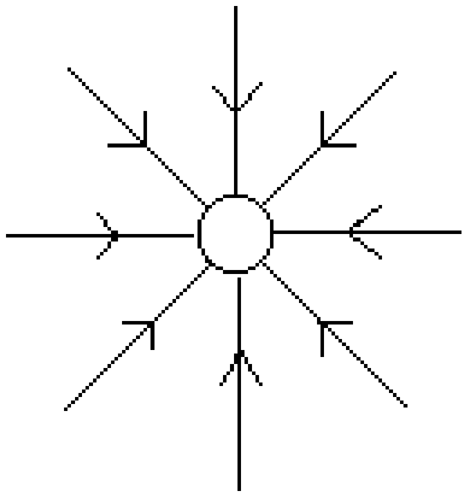
ELECTRIC LINES OF FORCE

- Are imaginary lines of force such that the tangent to it at any point gives the direction of electric field at that point.
- A positive point charge free to move will move in the direction of electric field and a negative point charge will move in a direction opposite to the direction of electric field along an electric line of force.

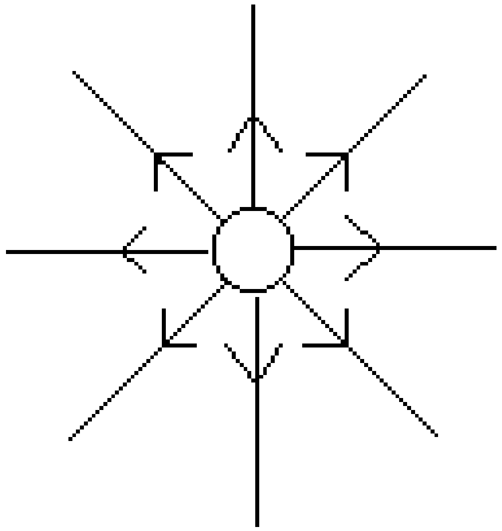
The lines of force to represent uniform electric field are as shown below



The electric lines of force due to point charge $q < 0$ are as shown below



The electric lines of force due to point charge $q > 0$ are as shown below



PROPERTIES OF ELECTRIC LINES OF FORCE

- Start from a positive charge and end in a negative charge.
- The tangent to it at any point gives the direction of electric field at that point.
- They never intersect each other
- They tend to contract longitudinally and expand laterally.
- They always enter or emerge normal to the surface of a charged conductor.
- They are close together in regions of strong electric field and far apart in regions of weak electric field.

Electric Field

Both Lori and the
spherical dome of the
Van de Graaff generator
are electrically charged.

ELECTRIC FLUX

Is the total lines of force passing normal to a given surface

$$\phi_E = \int_s \vec{E}.d\vec{S}$$

ϕ_E = E A for uniform electric field

Electric flux is a scalar quantity

2.2.1 (3)

Since in this model the fields strength is proportional to the number of lines per unit area, the *flux* of $\vec{E}$ ($\int \vec{E} \cdot d\vec{a}$) is proportional to the the number of field lines passing through any surface .

The flux of E through a sphere of radius r is:

$$\oint \vec{E} \cdot d\vec{a} = \int \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\hat{r}\right)\cdot\left(r^2 \sin\theta \, d\theta \, d\varphi \, \hat{r}\right) = \frac{1}{\varepsilon_0} q$$

The flux through any surface enclosing the charge is q/ε_0

According to the principle of superposition, the total field is the sum of all the individual fields:

$$\vec{E} = \sum_{i=1}^{n} \vec{E}_i \quad , \quad \oint \vec{E} \cdot d\vec{a} = \sum_{i=1}^{n}\left(\oint \vec{E}_i \cdot d\vec{a}\right) = \sum_{i=1}^{n}\left(\frac{1}{\varepsilon_0} q_i\right)$$

A charge outside the surface would contribute nothing to the total flux,since its field lines go in one side and out other.

GAUSS' THEOREM

States the total electric flux through a closed surface (surface integral of electric field over a closed surface) is equal to $1/\varepsilon_o$ times the total charge enclosed by the surface.

Mathematically

$$\oint_s \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}\left(q_{enclosed}\right)$$

2.2.1 (4)

Gauss's Law in integral form $\oint \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$

Turn integral form into a differential one , by applying the divergence theorem

$$\oint_{surface} \vec{E} \cdot d\vec{a} = \int_{volume} (\nabla \cdot \vec{E}) d\tau$$

$$= \frac{1}{\varepsilon_0} Q_{enc} = \int_{volume} (\frac{1}{\varepsilon_0} \rho) d\tau$$

Gauss's law in differential form $\nabla \cdot \vec{E} = \frac{1}{\varepsilon_0} \rho$

2.2.3 Application of Gauss's Law

Example 2.2 Find the field outside a uniformly charged sphere of radius a

Sol:

$$\oint_{surface} \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

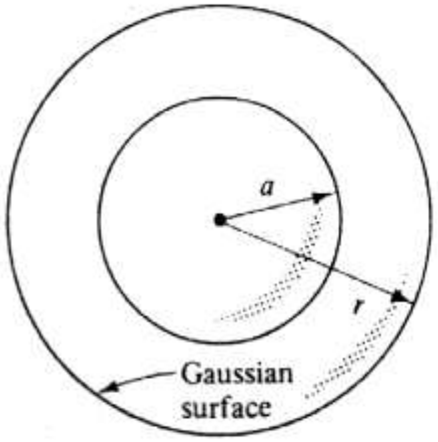


Figure 2.18

(a) E point radially outward ,as does $d\vec{a}$

$$\oint_{surface} \vec{E}\cdot d\vec{a} = \int |E| d\vec{a}$$

(b)E is constant over the Gaussian surface

$$\oint_{surface} |E| d\vec{a} = |E| \oint_{surface} d\vec{a} = |E| 4\pi r^2$$

Thus $|E| 4\pi r^2 = \frac{1}{\varepsilon_0} q \Longrightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

2.2.3 (2)

1. Spherical symmetry. *Make your Gaussian surface a concentric sphere (Fig 2.18)*
2. Cylindrical symmetry. *Make your Gaussian surface a coaxial cylinder (Fig 2.19)*
3. Plane symmetry. *Use a Gaussian surface a coaxial the surface (Fig 2.20)*

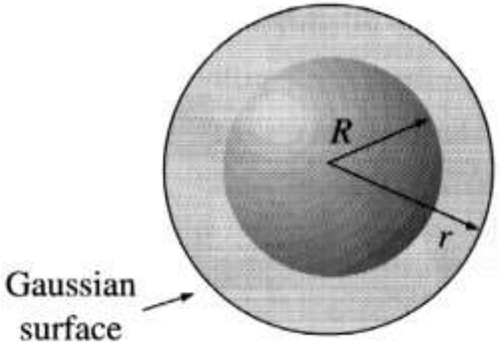


Figure 2.18

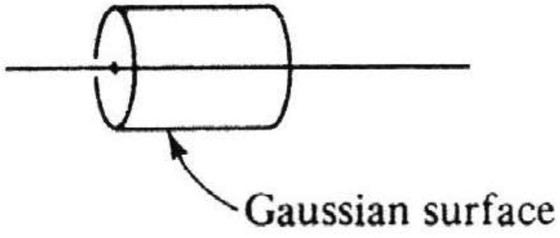


Figure 2.19

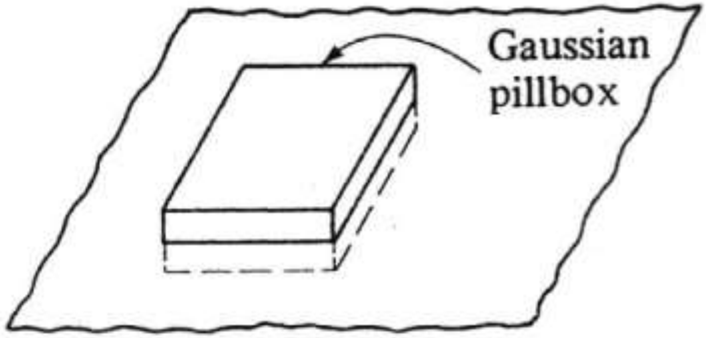


Figure 2.20

2.2.3 (3)

Example 2.3 Find the electric field inside the cylinder which contains charge density as $\rho = kr$

Solution:

$$\oint_{surface} \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

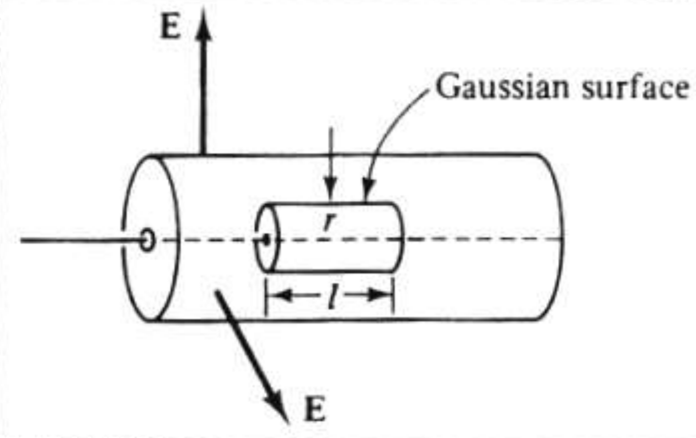


Figure 2.21

The enclosed charge is

$$Q_{enc} = \int \rho d\tau = \int (kr')(r'dr'd\varphi dz) = 2\pi k l \int_0^r r'^2 dr' = \frac{2}{3}\pi k l r^3$$

$$\oint \vec{E} \cdot d\vec{a} = \int |E| da = |E| \int da = |E| 2\pi r l$$ (by symmetry)

thus

$$|E| 2\pi r l = \frac{1}{\varepsilon_0} \frac{2}{3} \pi k l r^3 \Longrightarrow \boxed{\vec{E} = \frac{1}{3\varepsilon_0} k r^2 \hat{r}}$$

2.2.3 (4)

Example 2.4 An infinite plane carries a uniform surface charge. σ
Find its electric field.

Solution: Draw a "Gaussian pillbox
Apply Gauss's law to this surface

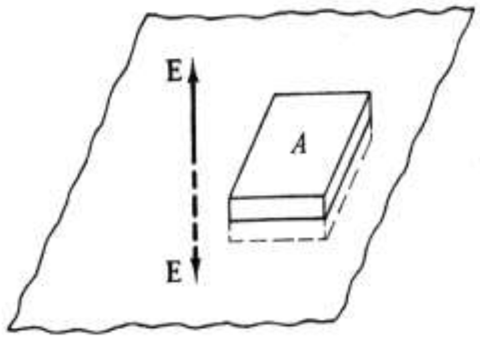


Figure 2.22

$$\oint_{surface} \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

By symmetry, E points away from the plane
thus, the top and bottom surfaces yields

$$\int \vec{E} \cdot d\vec{a} = 2A|E|$$

(the sides contribute nothing)

$$2A|E| = \frac{1}{\varepsilon_0} \sigma A \Longrightarrow \boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{n}}$$

2.2.3 (5)

Example 2.5 Two infinite parallel planes carry equal but opposite uniform charge densities $\pm\sigma$ .Find the field in each of the three regions.

Solution:

The field is (σ/ε_0), and points to the right, between the plane elsewhere it is zero.

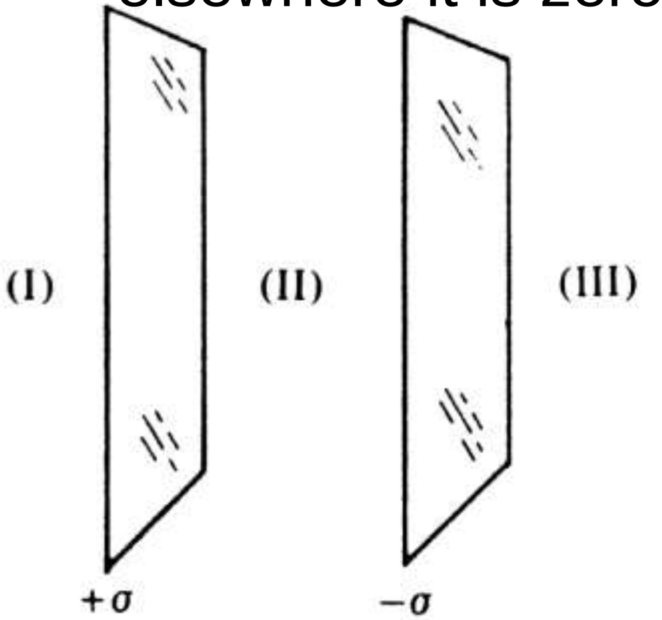


Figure 2.23

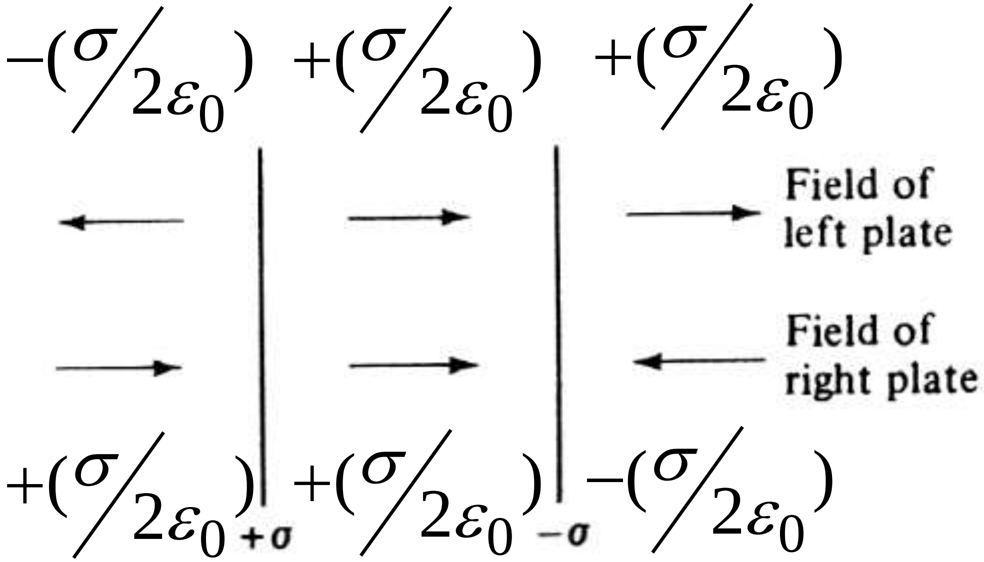


Figure 2.24

2.3.1 introduction to potential

Any vector whose curl is zero is equal to the gradient of some scalar. We define a function:

$$V(p) = -\int_{\vartheta}^{p} \vec{E} \cdot d\vec{l}$$

Where ϑ is some standard reference point ; *V* depends only on the point *P*. *V* is called the *electric potential*.

$$V(b) - V(a) = -\int_{\vartheta}^{b} \vec{E} \cdot d\vec{l} - \left(-\int_{\vartheta}^{a} \vec{E} \cdot d\vec{l} \right) = -\int_{a}^{b} \vec{E} \cdot d\vec{l}$$

The fundamental theorem for gradients

$$V(b) - V(a) = \int_{a}^{b} (\nabla V) \cdot d\vec{l}$$

so $$\int_{a}^{b} (\nabla V) \cdot d\vec{l} = -\int_{a}^{b} \vec{E} \cdot d\vec{l} \Longrightarrow \boxed{\vec{E} = -\nabla V}$$

2.3.2 Comments on potential

(1)The name

Potential is not potential energy

$$\vec{F} = q\vec{E} = -q\nabla V \qquad \Delta U = \vec{F} \cdot \vec{X}$$

V : Joule/coulomb $\quad$ U : Joule

2.3.2 (2)

(2)Advantage of the potential formulation

V is a scalar function, but E is a vector quantity

$$V(r) \qquad\qquad \vec{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$$

If you know *V*, you can easily get *E:* $\vec{E} = -\nabla V$.

E_x, E_y, E_z are not independent functions

$$\nabla \times \vec{E} = 0 \quad \text{so} \quad \frac{\partial E_x}{\partial y} \overset{①}{=} \frac{\partial E_y}{\partial x}, \quad \frac{\partial E_z}{\partial y} \overset{②}{=} \frac{\partial E_y}{\partial z}, \quad \frac{\partial E_x}{\partial z} \overset{③}{=} \frac{\partial E_z}{\partial x}$$

$$\partial_y \partial_z E_x = \partial_z \partial_y E_x \overset{①}{=} \partial_z \partial_x E_y = \partial_x \partial_z E_y \overset{②}{=} \partial_x \partial_y E_z = \partial_y \partial_x E_z \Rightarrow \partial_z E_x \overset{③}{=} \partial_x E_z$$

2.3.2 (3)

(3)The reference point ϑ

Changing the reference point amounts to adds a constant to the potential

$$V'(p) = -\int_{\vartheta'}^{p} \vec{E} \cdot d\vec{l} = -\int_{\vartheta'}^{\vartheta} \vec{E} \cdot d\vec{l} - \int_{\vartheta}^{p} \vec{E} \cdot d\vec{l} = K + V(p)$$

(Where K is a constant)

Adding a constant to *V* will not affect the potential difference between two point:

$$V'(b) - V'(a) = V(b) - V(a)$$

Since the derivative of a constant is zero: $\nabla V = \nabla V'$
For the different *V*, the field *E* remains the same.

Ordinarily we set $V(\infty) = 0$

2.3.2 (4)

(4)Potential obeys the superposition principle

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots \quad , \quad \vec{F} = Q\vec{E}$$

Dividing through by Q

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots$$

Integrating from the common reference point to p ,

$$V = V_1 + V_2 + \cdots$$

(5)*Unit of potential*

Volt=Joule/Coulomb

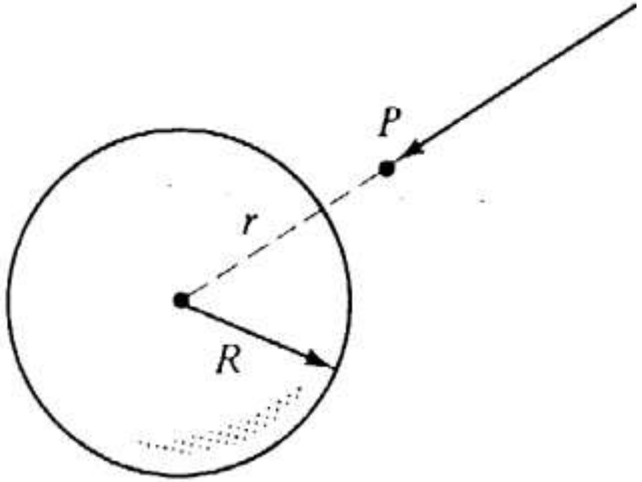
$$F : newton \qquad F \cdot x : Joule$$

$$F = qE = q\nabla V \rightarrow \frac{qV}{X}$$

$V : \dfrac{F \cdot X}{q} \rightarrow$ Joule/Coulomb

2.3.2 (5)

Example 2.6 Find the potential inside and outside a spherical shell of radius R, which carries a uniform surface charge (the total charge is q).



solution:

$$\vec{E}_{in} = 0 \qquad \vec{E}_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

for r>R:

$$V(\vec{r}) = -\int_{\infty}^{r} \vec{E}\cdot d\vec{\ell} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r'}\bigg|_{\infty}^{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

for r≤R:

$$V(R) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \qquad V(\vec{r}) \neq V(R) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

↑

$$\vec{E}_{in} = 0$$

2.3.3 Poisson's Eq. & Laplace's Eq.

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = \nabla \cdot \left(-\nabla V\right) = -\nabla^2 V$$

↑

$$\vec{E} = -\nabla V$$

Poisson's Eq. $\boxed{\nabla^2 V = -\frac{\rho}{\varepsilon_0}}$

$\rho = 0$ $\boxed{\nabla^2 V = 0}$ Laplace's eq.

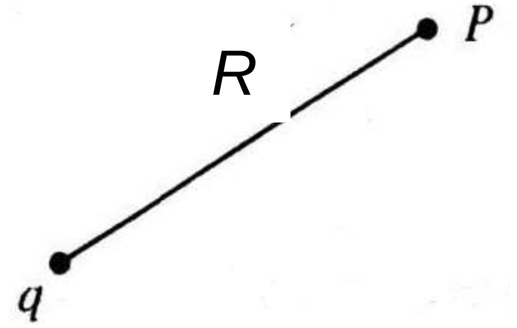
2.3.4 The Potential of a Localized Charge Distribution

$$\vec{E} = -\nabla V \qquad V - V_\infty = -\int_\infty^r E dr' \qquad V_\infty = 0$$

$$V(r) = \frac{-1}{4\pi\varepsilon_0}\int_\infty^r \frac{q}{r'^2}dr' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r'}\bigg|_\infty^r = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

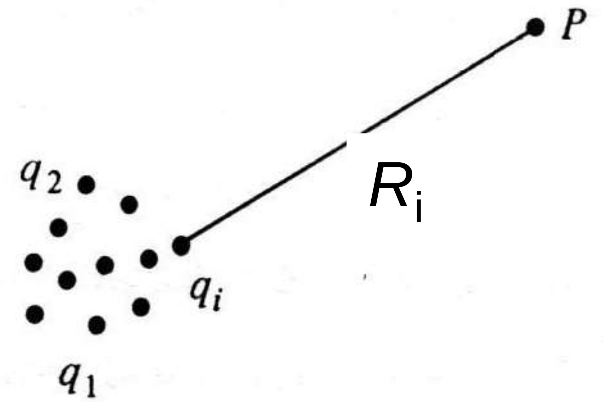
- Potential for a point charge

$$V(P) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \qquad R = \left|\vec{r} - \vec{r}_p\right|$$



- Potential for a collection of charge

$$V(P) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{R_i} \qquad R_i = \left|\vec{r}_i - \vec{r}_p\right|$$



2.3.4 (2)

- Potential of a continuous distribution

for volume charge

$$\delta q = \rho d\tau$$

$$V(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{R} d\tau$$

for a line charge

$$\delta q = \lambda d\ell$$

$$V(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda}{R} d\ell$$

for a surface charge

$$\delta q = \sigma da$$

$$V(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma}{R} da$$

- **Corresponding electric field** $\left[\ -\nabla \frac{1}{R} = \frac{\hat{R}}{R^2}\ \right]$

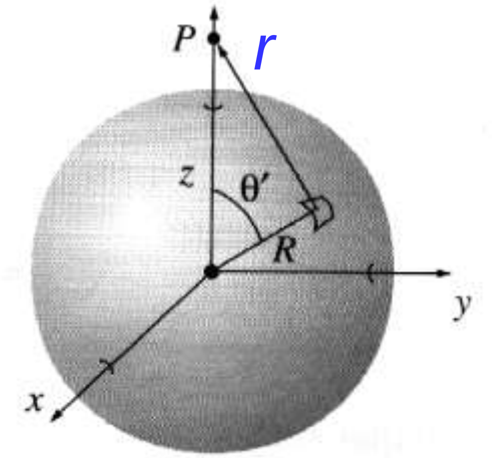
$$\vec{E}(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{r}}{R^2}\rho d\tau \quad \vec{E}(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{r}}{R^2}\lambda d\ell \quad \vec{E}(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{r}}{R^2}\sigma\, da$$

2.3.4 (3)

Example 2.7 Find the potential of a uniformly charged spherical shell of radius *R*.

Solution:

$$V(r)=\frac{1}{4\pi\varepsilon_0}\int\frac{\sigma}{r}da', \qquad r^2=R^2+z^2-2Rz\cos\theta'$$



$$4\pi V(z)=\sigma\int\frac{R^2\sin\theta' d\theta' d\varphi'}{\sqrt{R^2+z^2-2Rz\cos\theta'}}$$

$$=2\pi R^2\sigma\int_0^\pi\frac{\sin\theta'}{\sqrt{R^2+z^2-2Rz\cos\theta'}}d\theta'$$

$$=2\pi R^2\sigma\left(\frac{1}{Rz}\sqrt{R^2+z^2-2Rz\cos\theta'}\right)\Big|_0^\pi$$

$$=\frac{2\pi R\sigma}{z}\left(\sqrt{R^2+z^2+2Rz}-\sqrt{R^2+z^2-2Rz}\right)$$

$$=\frac{2\pi R\sigma}{z}\left[\sqrt{(R+z)^2}-\sqrt{(R-z)^2}\right]$$

2.3.4 (4)

$$V(z) = \frac{R\sigma}{2\varepsilon_0 z}[(R+z)-(z-R)] = \frac{R^2\sigma}{\varepsilon_0 z}, \quad outside$$

$$V(z) = \frac{R\sigma}{2\varepsilon_0 z}[(R+z)-(R-z)] = \frac{R\sigma}{\varepsilon_0}, \quad inside$$

- Is the **surface** of a **charged conductor** an equipotential?

- Is the electric potential constant everywhere **inside** a charged conductor and equal to its value at the surface?

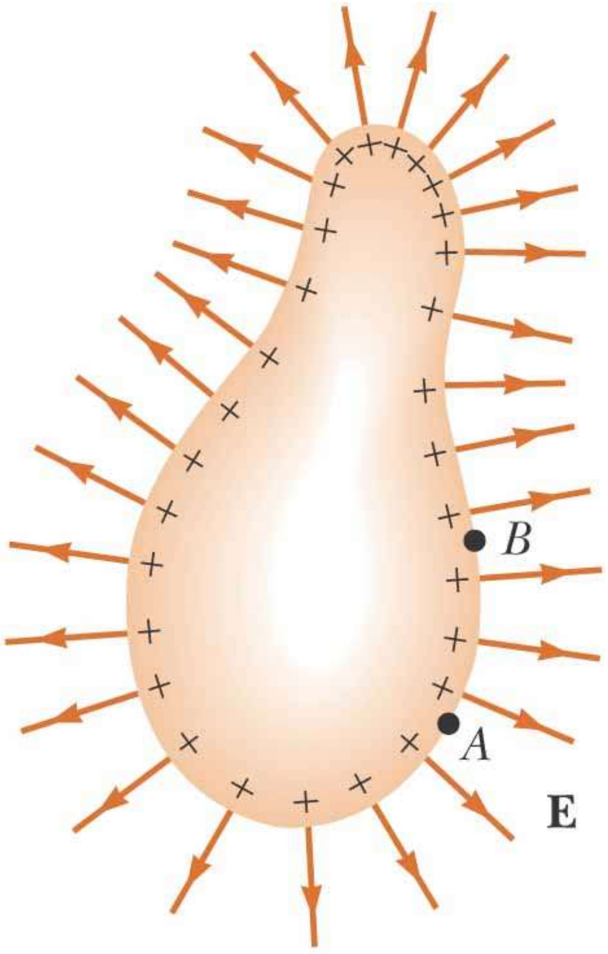
Electric Potential Difference on the Surface of a Charged Conductor in Equilibrium

- Let A and B be points on the surface of the charged conductor
- Let ds be the displacement from A to B.
- E is always perpendicular to the displacement $d\mathbf{s}$.

  So, $\mathbf{E} \cdot d\mathbf{s} = 0$

- Therefore, the potential difference between A and B is also zero

$$\Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$



Electric Potential Difference on the Surface of a Charged Conductor in Equilibrium

- V is constant everywhere on the surface of a charged conductor in equilibrium

- $\Delta V = 0$ between any two points on the surface

- The surface of any charged conductor in electrostatic equilibrium is an equipotential surface

What about the inside of a charged conductor?

- E=0 inside the conductor in equilibrium
- $\mathbf{E} \cdot d\mathbf{s} = 0$

$$\Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

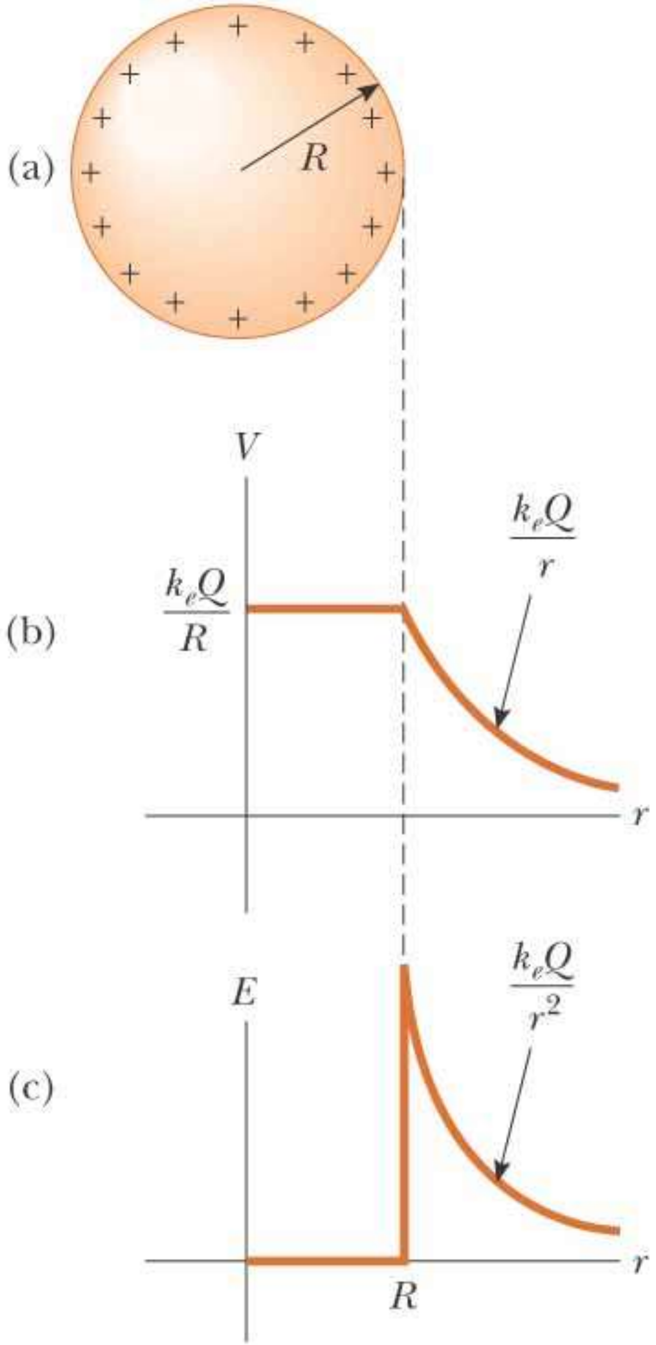
- Therefore, the electric potential is constant everywhere inside the conductor and equal to the value at the surface.

Solid Conducting Sphere

- r<R V=kq/R E=0
- r=R V=kq/R $E=kQ/R^2$
- r>R V=kq/r $E=kQ/r^2$

Note:

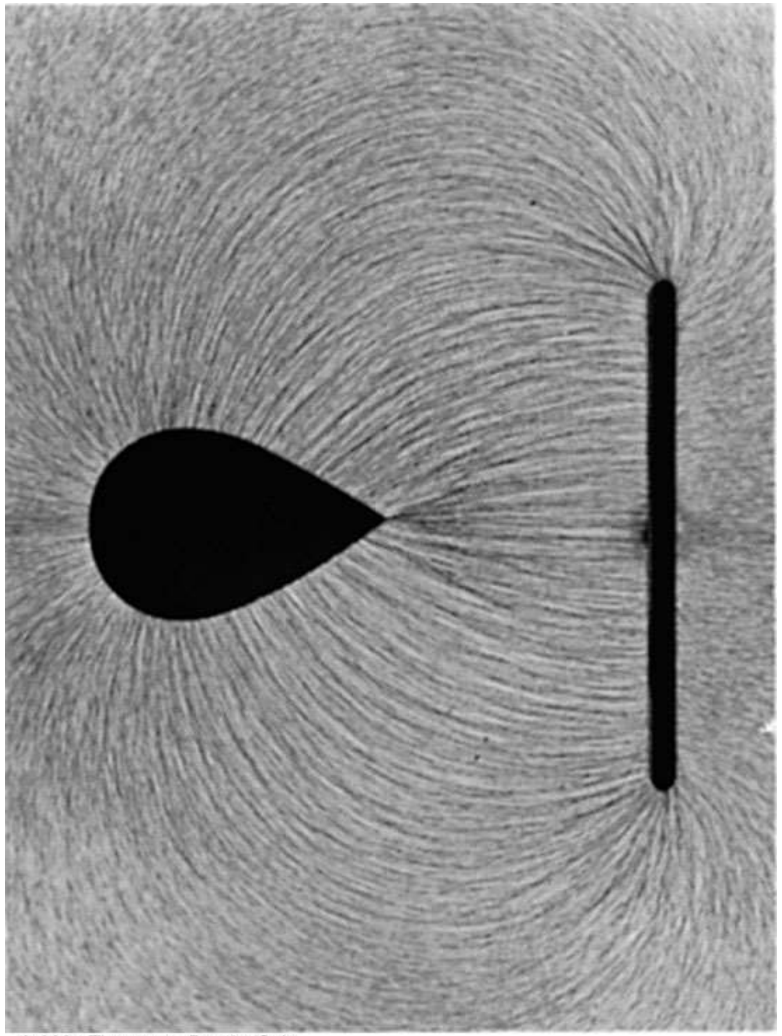
- V is a Scalar related to energy
- E is a Vector related to force.



Irregularly Shaped Conductors

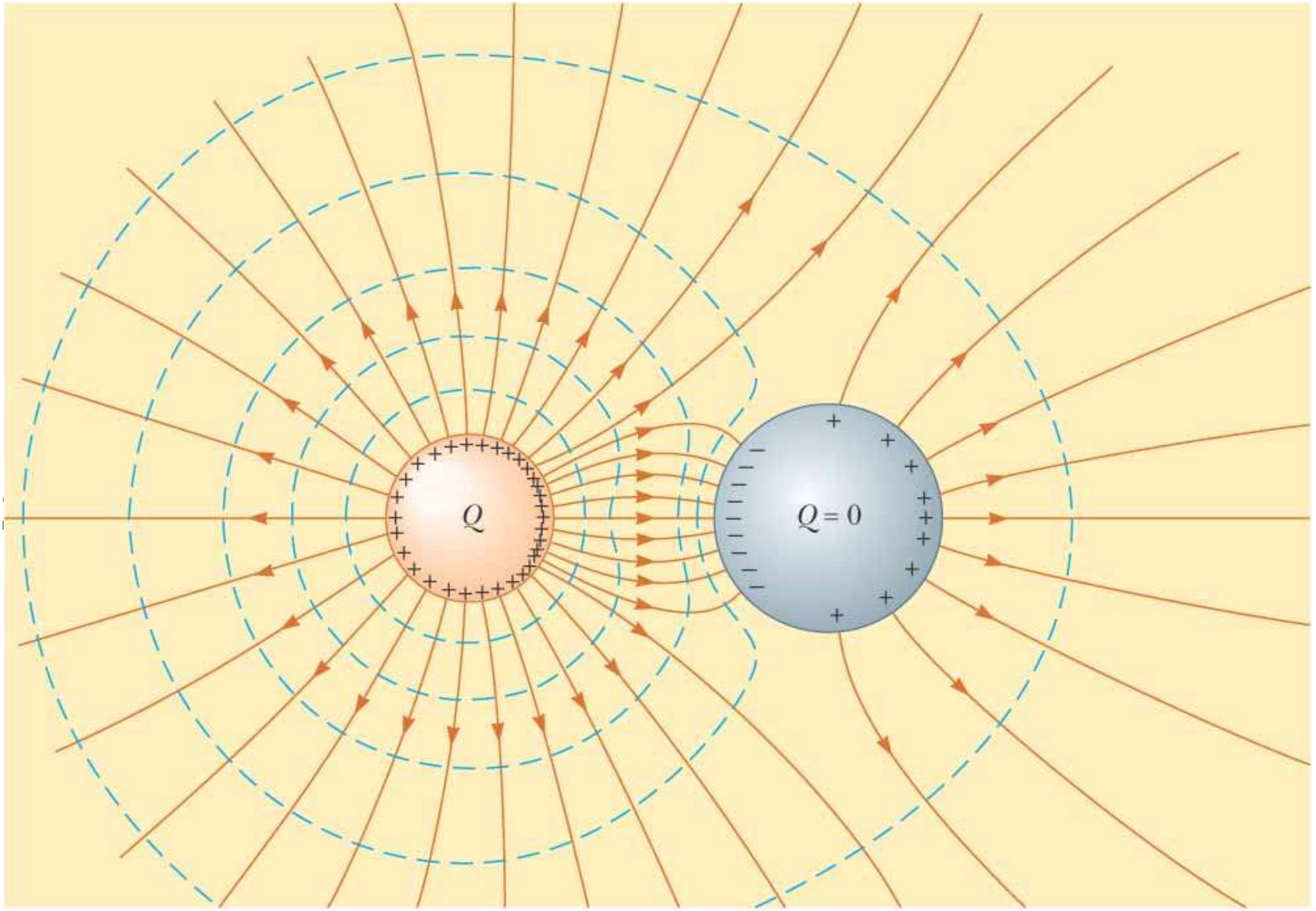
- The charge density is high where the radius of curvature is small

- The electric field is high at sharp points



Irregularly Shaped Conductors

- The field lines are perpendicular to the conducting surface
- The equipotential surfaces are perpendicular to the field lines



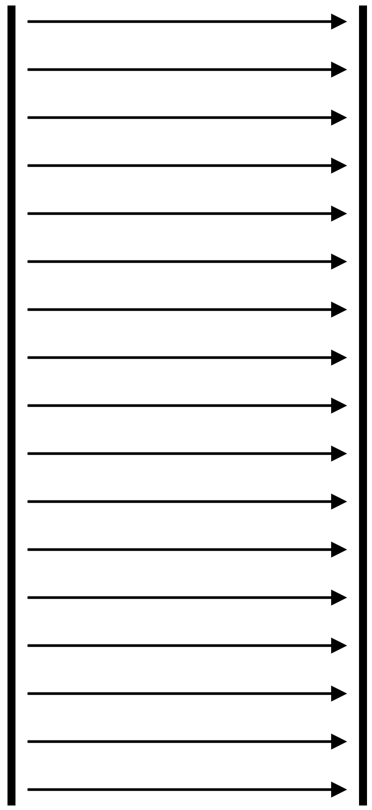
Electric Potential –What we used so far!

- Electric Potential $$V = \frac{U}{q_o}$$
- Potential Difference $$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$
- Potential for a point charge $$V = k_e \frac{q}{r}$$
- Potential for multiple point charges $$V = k_e \sum_i \frac{q_i}{r_i}$$
- Potential for continuous charge distribution $$V = k_e \int \frac{dq}{r}$$

PRINCIPLE OF A CAPACITOR

- Capacitor is based on the principle that the capacitance of an isolated charged conductor increases when an uncharged earthed conductor is kept near it and the capacitance is further increased by keeping a dielectric medium between the conductors.

CAPACITANCE OF A PARALLEL PLATE CAPACITOR



Electric field between the plates,

$E = \sigma/\varepsilon_0$

But $\sigma=Q/A$

$\therefore E=Q/A\varepsilon_0$

Potential difference between the two plates , $V = Ed = Qd/A\ \varepsilon_0$

Capacitance, $C = Q/V$

$C=A\ \varepsilon_0/d$

CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH A DIELECTRIC SLAB

When a dielectric slab is kept between the plates **COMPLETELY** filling the gap

$E' = E_0/K$ where K is the dielectric constant of the medium.

Potential difference

$V' = E'd = E_0 d/K = Qd/K\varepsilon_0 A$

Capacitance $C' = Q/V' = K\varepsilon_0 A/d = KC$

$\therefore$ when a dielectric medium is filled between the plates of a capacitor, its capacitance is increased K times.

DIELECTRIC STRENGTH

- Dielectric strength of a dielectric is the maximum electric field that can be applied to it beyond which it breaks down.